The Silver Rectangle / Silver Triangle

The silver rectangle has sides in the proportion of 1:Sqrt2 and its diagonals cross at angles of 70.32 and 109,28 degrees. The term silver rectangle was first adopted as a suitable name for the or true rectangle by the British Origami Society in 1979, following a suggestion made by the science section of the Oxford Dictionary organisation, and has since become an established paperfolding term.

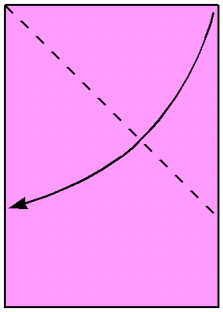
Somewhat confusingly, however, the same name is sometimes used, particularly in the USA, for the 1:1+Sqrt2 rectangle. I prefer, however, to refer to the 1:Sqrt2 rectangle as the silver rectangle and the 1:1+Sqrt2 rectangle as the leftover rectangle, since it is the piece leftover when you remove the largest possible square from a 1:Sqrt2 rectangle.

The 1:Sqrt2 rectangle is also sometimes known to mathematicians as the true rectangle. The angle of 109,28 degrees is sometimes known as the Miraldi Angle.

Standard DIN paper sizes, A4, A5, A6 etc, common throughout the world, except in the USA, are very close to being silver rectangles, the small differences being due to the various sizes being specified in whole millimetres and to cutting variances during manufacture. If you find that you are using DIN paper that is sufficiently out of proportion to affect the folding geometry you can usually diminish the error by folding it short edge to short edge and cutting it in half.

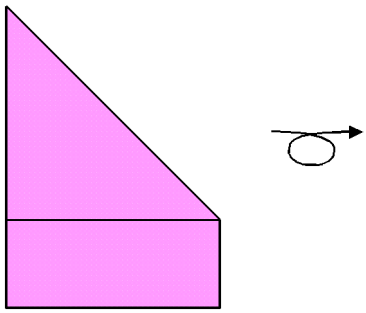
You can check if a sheet of paper is an accurate silver rectangle like this:

1



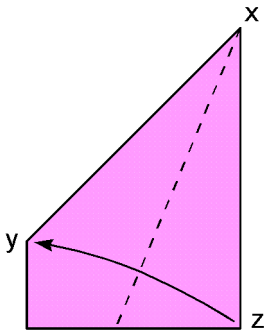
1. Fold one of the short edges onto one of the long edges.

2



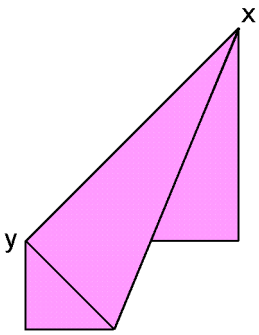
2. Turn over sideways.

3



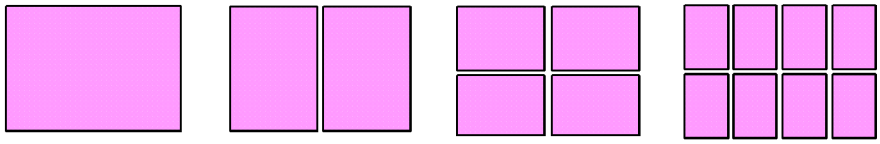
3. Fold the right hand edge onto the sloping edge so that point x remains sharp.

4

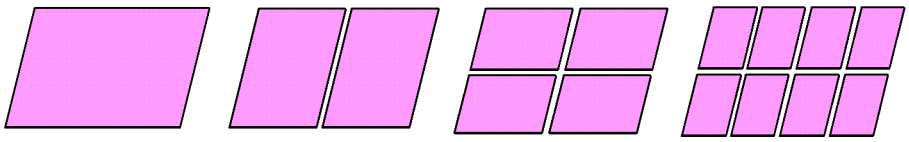


4. If your paper is a silver rectangle points y and z will lie exactly on top of each other.

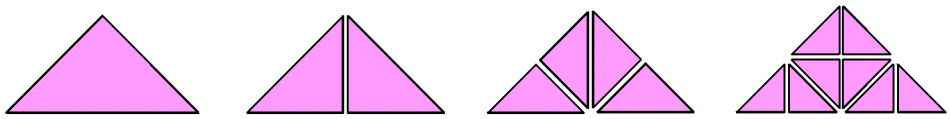
Folding a silver rectangle in half short edge to short edge and cutting along the crease produces two smaller silver rectangles. This process can be repeated ad infinitum.



Any parallelogram with long and short sides in the same proportion possesses the same property.

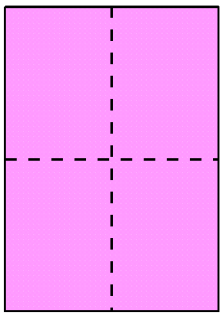


The only other shape that possesses this property is the 1:1:Sqrt2 isosceles triangle made by cutting a square diagonally in half. For this reason this triangle can be referred to as the silver triangle.



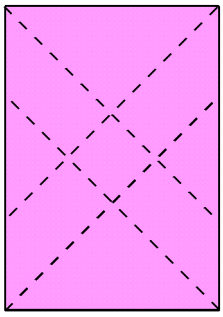
Primary folding geometry

1



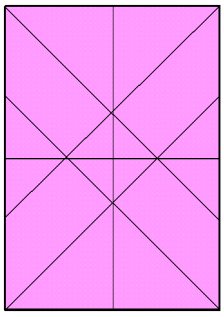
1. The silver rectangle can be folded edge to opposite edge in both directions.

2



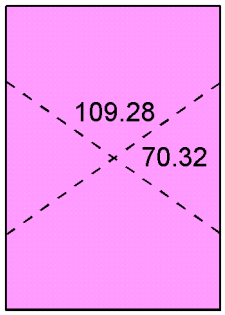
2. Each short edge can be folded onto both long edges.

3



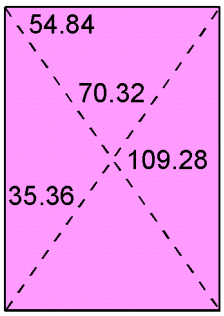
3. This is the standard folding geometry of the silver rectangle. This folding geometry is common to all rectangles except the square.

4



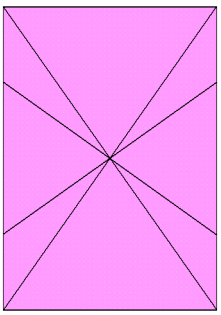
4. Folding corner to opposite corner produces these creases.

5



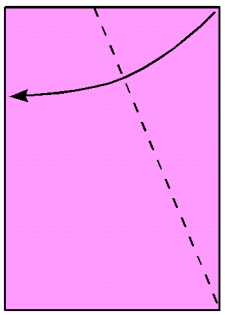
5. Diagonals can be added by creasing between opposite corners.

6



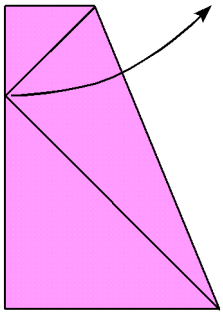
6. The angles at which the diagonals cross are particularly significant for the folding geometry. In this case the angles of 70.32 and 109,28 degrees (otherwise sometimes known as the Miraldi angle) occur in the rhombic polyhedra and in nolid / planar cubes and tetrahedra. This can be referred to as silver folding geometry.

7



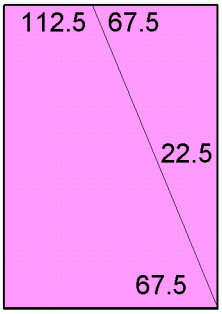
7. A third primary folding geometry can be derived by folding the top right corner onto the opposite long edge while making sure that the crease passes through the bottom right hand corner.

8



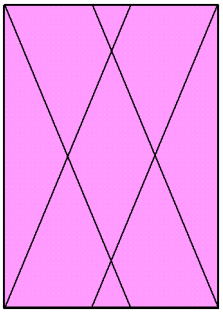
8. The result should look like this.

9



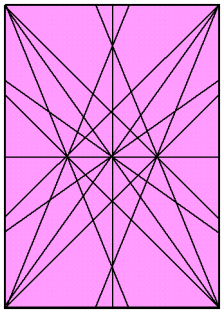
9. This yields a folding geometry based on 22.5 and 67.5 degrees. For reasons which will become apparent later on we can refer to this as leftover rectangle geometry.

10



10. All four corners can be folded inwards inn the same way.

11

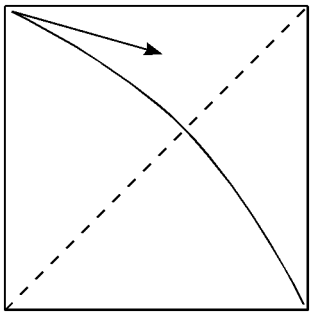


11. The combined ;primary folding geometries of the silver rectangle look like this. All rectangles other than the square show a similar pattern although the angular systems vary.

Constructing silver rectangles

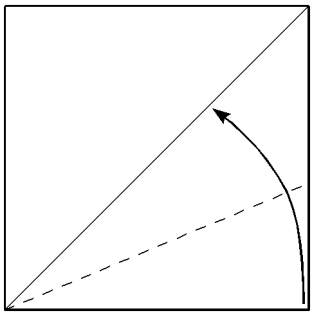
Method 1 - from the square

1



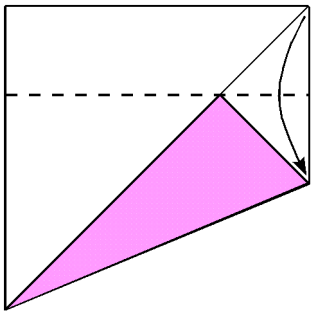
1. Fold a square in half diagonally, then unfold.

2



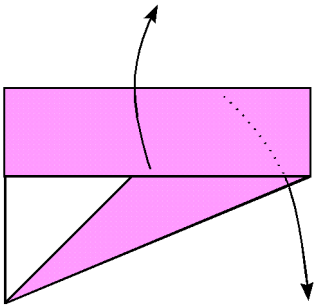
2. Fold the bottom edge onto the diagonal crease.

3



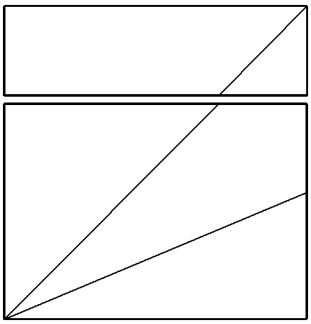
3. Fold the right hand edge in half downwards.

4



4. Open out.

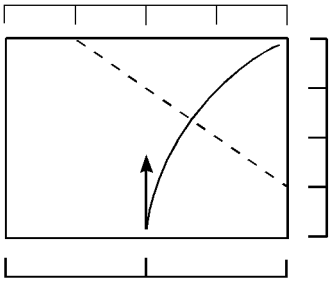
5



5. Separate the pieces by cutting along the horizontal crease. The larger piece is a silver rectangle.

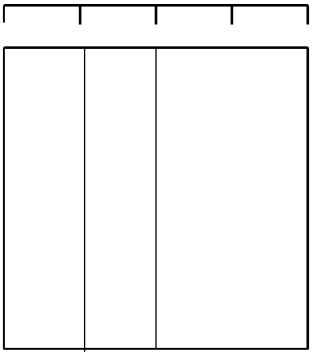
Method 2 - From any rectangle

It is a property of the silver rectangle that folding any corner to the centre of the opposite long edge produces a crease that passes through the three-quarter-way distant point of the two edges adjacent to that corner.



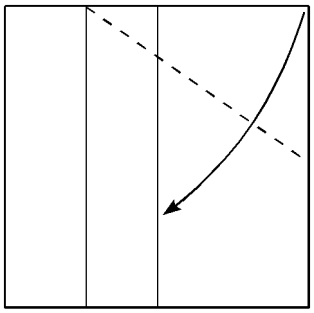
This useful property can be used (in reverse) to construct a silver rectangle from other rectangle, including the square.

1



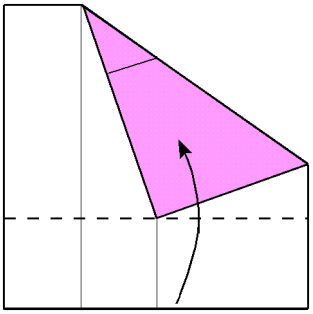
1. Begin by making two vertical creases like this at the halfway and quarter-way points.

2



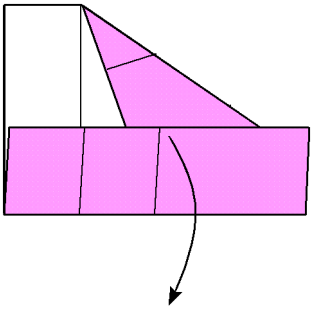
2. Fold the top right hand corner onto the central vertical crease making sure that the new crease starts at the point where the quarter-way crease intersects the top edge.

3



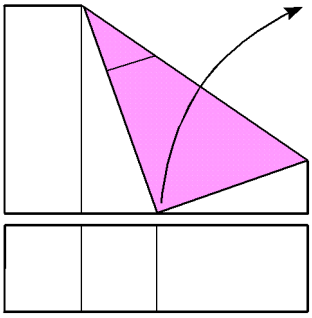
3. Fold the bottom edge upwards using the bottom point of the front layer to locate the fold.

4



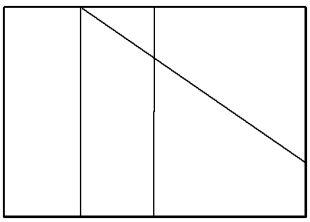
4. Open out.

5



5. Separate the pieces by cutting along the horizontal crease.

6



6. This piece is a silver rectangle.