## The leftover rectangle

The leftover rectangle, for the name of which I always use all lower case letters, because of its humble status, has sides in the proportion of $1: 1+$ Sqrt2 and diagonals that cross at angles of $45 / 135$ degrees. It is the rectangle that is leftover if you remove the largest possible square from a silver rectangle.

## 1



1. In paperfolding, the leftover rectangle is most commonly encountered as the rectangle that is left over ...

2. ... when the largest possible square is removed from a silver rectangle.

## 4


4. ... the piece left over is a 1 :sqrt2 or silver rectangle.

6. ... ad infinitum.

Four leftover rectangles can be laid over each to produce an octahedron, like this.


Primary folding geometry
The primary folding geometry of the leftover rectangle can be constructed in the same way as for the silver rectangle (or any rectangle other than a square ). The diagonals cross at angles of 45 and 135 degrees as shown below.

The long diagonals intersect the top and bottom edges at an angle of 22.5 degrees. We can use this to construct leftover rectangles from rectangles other than the silver rectangle.


## Constructing leftover rectangles

From the silver rectangle

Method 1 creates a leftover rectangle by removing the largest possible square from the rectangle. This is the method for which the leftover rectangle is named. Method 2 creates the largest possible leftover rectangle from a given size of silver rectangle. Methods 3 and 4 are elegant ways of obtaining multiple leftover rectangles from one silver rectangle.

## Method 1



1. Fold the top edge onto the left edge, then unfold.

2. Fold the bottom edge upwards as shown, then unfold.

3. Separate the leftover rectangle from the square by cutting along the crease made

## Method 2

1


1. Fold one short edge onto one long edge, then unfold.

2

2. Fold a long edge onto the diagonal crease, then unfold.

3. Fold the opposite long edge inwards, using the point where the crease made in step 2 intersects the bottom edge to locate the fold, then unfold.

4. Separate the pieces by cutting along the vertical crease.

## Method 3

1


1. Fold in half sideways, then unfold.
3

2. Fold the top right corner inwards like this, then unfold.

3. The larger piece is a leftover rectangle. This is the largest leftover rectangle it is possible to cut from any given size of silver rectangle.
2

4. Fold in half downwards, then unfold.

4

4. Fold the top edge downwards like this, then unfold.

David Mitchell / The leftover rectangle

5

5. separate the pieces by cutting along the creases marked with thick lines.
Both rectangles are the same size.

## Method 4

1


1. Fold the top left corner onto the right edge making sure the crease starts from the bottom left corner, then unfold.

3

3. Fold the bottom point upwards like this, then unfold.

2

2. Fold the top edge onto the left edge, then unfold.

4

4. Fold the right edge inwards like this, then unfold.
5

6

65. Fold the left edge inwards like this, then unfold.
6. Separate the pieces by cutting along the creases marked with thick black lines. All three rectangles are the same size.

## From the square



1. Fold a square in half diagonally, then unfold.

2. Fold the bottom edge onto the diagonal crease, then unfold.

3

3. Fold the bottom edge inwards ( using the point marked with a circle to locate the fold ), then unfold.

4. Cut along the line of the crease made in step 3. The smaller piece is a leftover rectangle.
5. If you repeat steps 2 and 3 on the top half of the paper ...
6


6. ... you can make two leftover rectangles of the same size from a single square.

