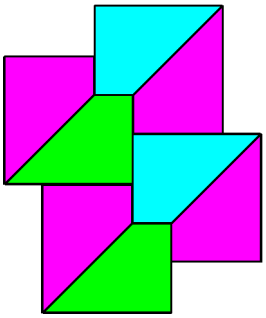
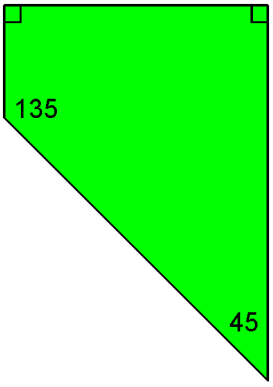
Trapezium Tiles

The shape of the Trapezium Tile is just a rectangle from which one corner has been removed (by folding it away rather than by cutting it off). It is the simplest of all origami tiles and therefore a good place to begin.



You can create a trapezium tile by removing a corner from any rectangle except the square. The angles of all such tiles will be the same, but the proportions of the sides will vary.



These diagrams show you how to fold Trapezium tiles from silver rectangles, of which DIN shape rectangles, A4, A5 etc are good enough approximations for paperfolding purposes. There are two advantages to using this shape of paper. Firstly it produces a tile whose two longer edges are both the same length, which makes possible the pattern shown in picture 7. Secondly it produces a tile whose perimeter can be shown to be three times the length of the long edge of the starting rectangle, or 3xsqrt2 (see notes).

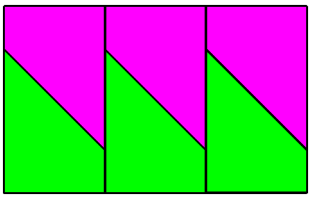
Because the Trapezium tile is asymmetric, mirror image tiles are possible. These are easily obtained by varying the folding sequence as shown in steps 4 to 6 on page 4. Sets of mirror-image tiles are used to create the patterns shown in pictures 5 and 6.

Working out the internal angles of the Trapezium tile provides a very basic geometric exercise.

I first played with Trapezium tiles in 2010 and discovered the perimeter result at that time. I would be surprised to learn that this was not already known.

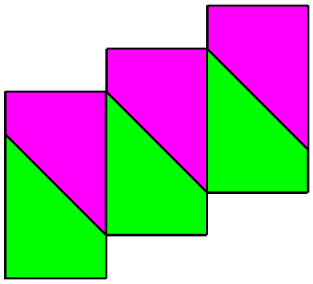
Tiling patterns

1



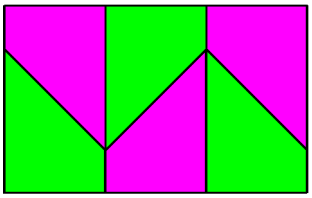
1. Two tiles fit together to form a rectangle These rectangles will tile the plane like this ...

2



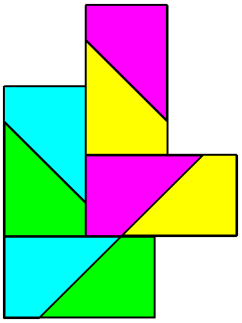
2. ... or like this.

3



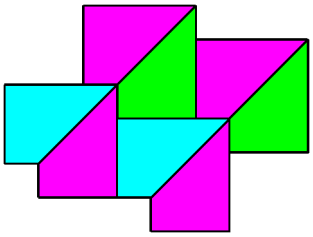
3. They will also tile the plane like this ...

4



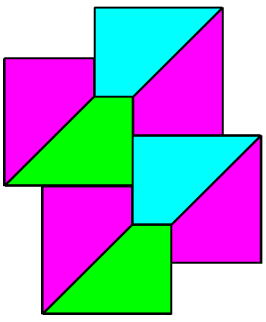
4. ... or like this.

5



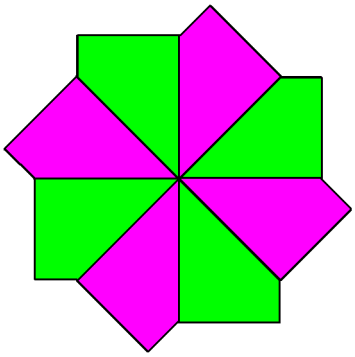
5. Two mirror image tiles will go together to form a square with an indented corner, which will tile the plane like this ...

6



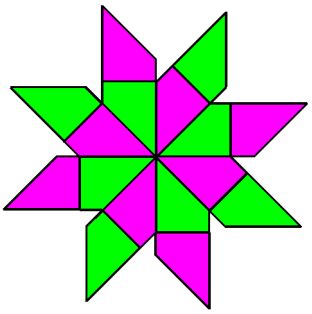
6. ... or like this.

7



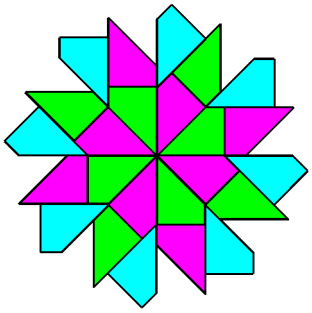
7. Trapezium tiles will also go together to form a rotational pattern like this.

8



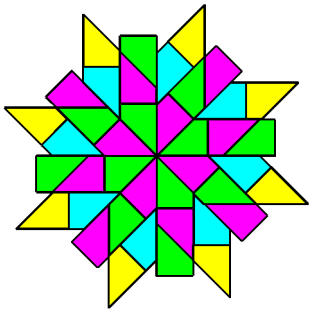
8. This pattern can be extended by adding extra tiles all sides.

9



9. If you continue adding tiles like this ...

10

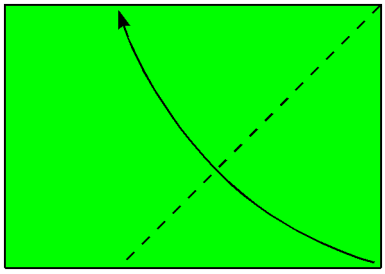


10. ... the pattern will extend outwards on all sides and tile the plane.

Folding Trapezium tiles

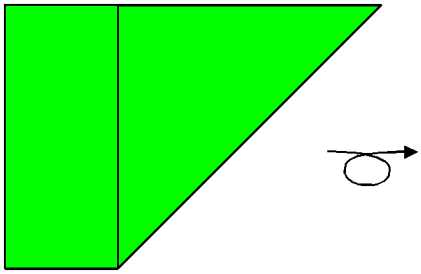
Trapezium tiles can be folded from any shape of rectangle other than the square.

1



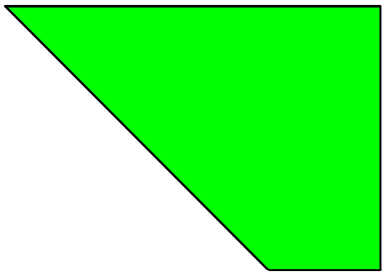
1. Fold the bottom edge onto the right hand edge.

2



2. Make sure the layers all lie flat then turn over sideways.

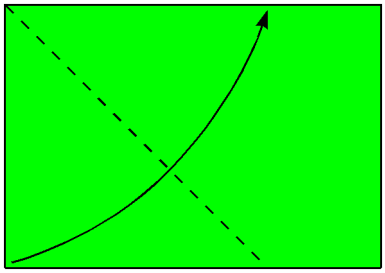
3



3. Finished.

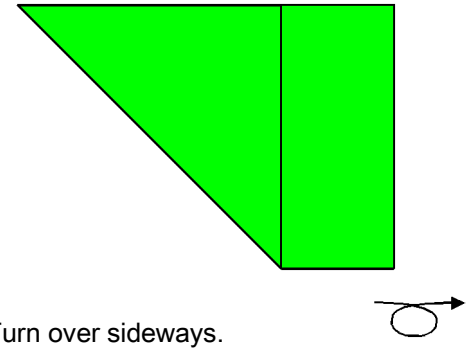
Folding mirror image tiles

4



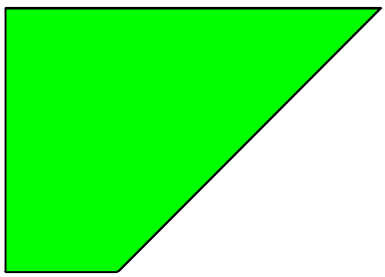
4. To make a mirror-image tile simply fold in the bottom right corner instead.

5



5. Turn over sideways.

6

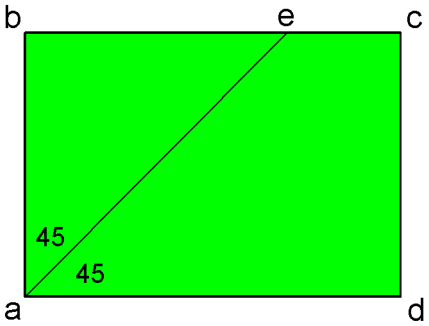


6. The mirror-image tile is finished.

Notes

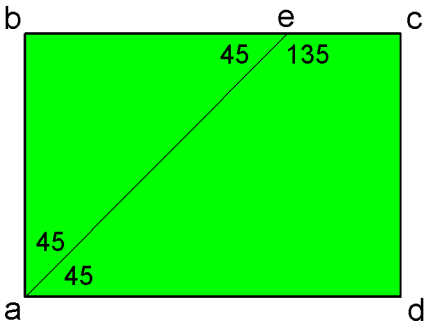
Working out the internal angles

A



A. Because crease ae (made in step 1) bisects corner a, which is a right angle, we know that the internal angle of the tile at corner a is 45 degrees.

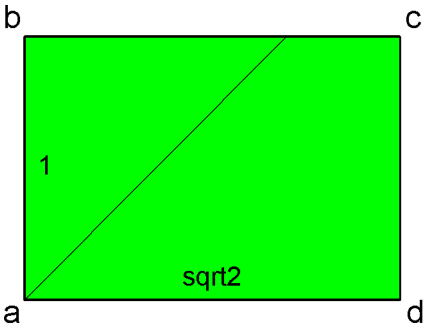
B



B. Because the top and bottom edges of a rectangle are parallel we know that crease ae must also intersect the top edge at 45 degrees. The internal angle at e is therefore 135 degrees. The angles at c and d are of course right angles. The angles of a trapezium sum to 360 degrees.

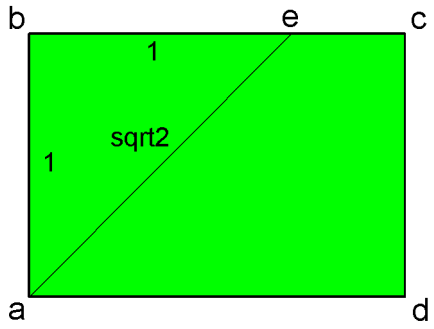
Calculating the perimeter

C



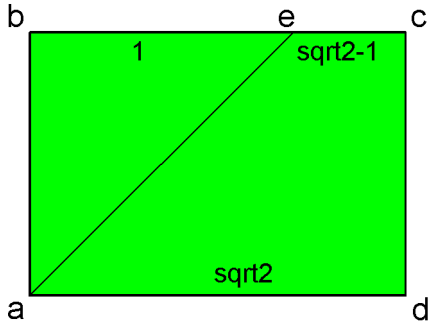
C. The starting shape is a silver rectangle with edges in the proportion 1:sqrt2.

D



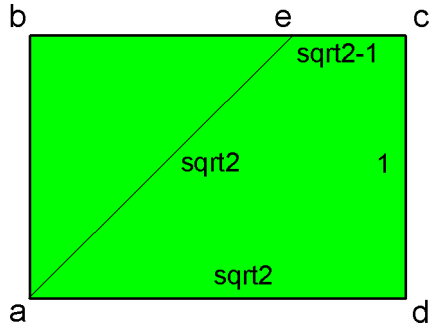
D. The area above and to the right of crease be is an isosceles triangle. Length be is therefore the same as length ab. By Pythagoras we know that length ae is therefore sqrt2.

E



E. Since we know that length bc is sqrt2 and be is 1 length ec must be sqrt2-1.

F



F. The perimeter of the tile is therefore 3 (sqrt2), three times the length of the long edge of the starting rectangle.