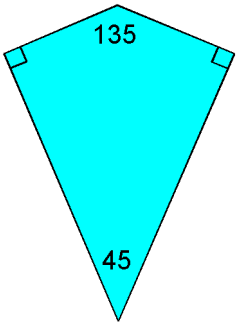
Octagon Kite Tiles

The octagon kite tile is easy to fold from both the square and the silver rectangle. Two methods of folding the tile from either shape are included in these diagrams.



Silver rectangles have edges in the proportion 1:sqrt2. DIN paper sizes such as A4, A5 etc are good enough approximations of silver rectangles for practical paperfolding purposes.

When folded from a silver rectangle using method 1 the perimeter of the octagon kite tile is four times the length of the short edge of the silver rectangle it is folded from.

Silver rectangles can easily be folded into other shapes that share this property. The Notes show you how to fold four of these, two rectangles, for which the location points are provided by the outline of the octagon kite tile, and two simple double trapezium tiles, both of which will tile the plane.

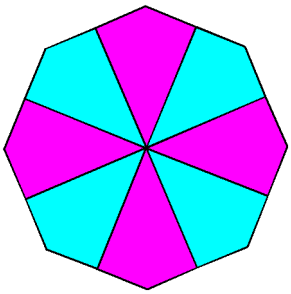
I do not know who first discovered how to make an octagon kite tile from a silver rectangle using method 1. I discovered method 2 in 2010.

I first learned that the perimeter of an octagon kite tile folded from a silver rectangle using method 1 is length 4 from Sue Pope. The two rectangles and the two double trapezium tiles are my own discoveries.

It is also worth noting that all versions of the octagon kite tile can easily be turned into Cairo tiles using just a single extra fold.

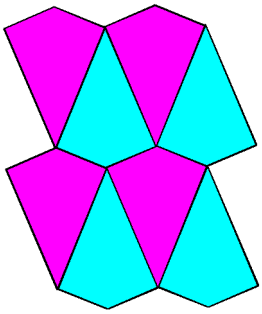
Tiling patterns

1



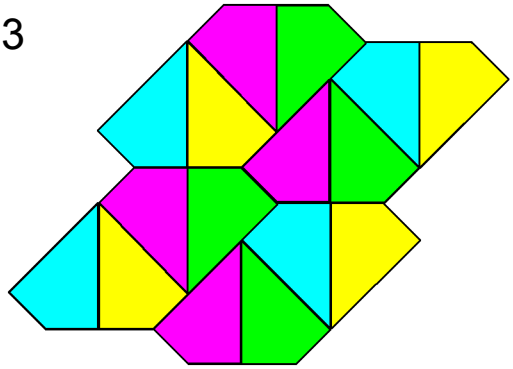
1. Eight tiles form an octagon.

2



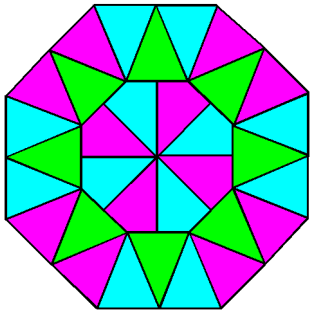
2. Rows of tiles will tile the plane.

3



3. You can also tile the plane like this

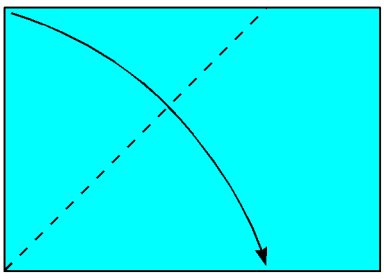
4



5. Octagon kite tiles and octagon Isosceles tiles, of the right scale, can be combined into a more complex octagon like this.

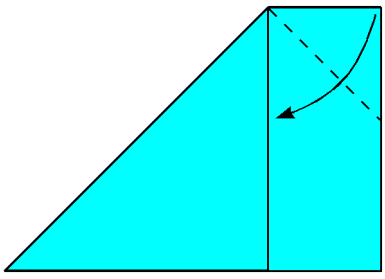
Folding from the silver rectangle - method 1

5



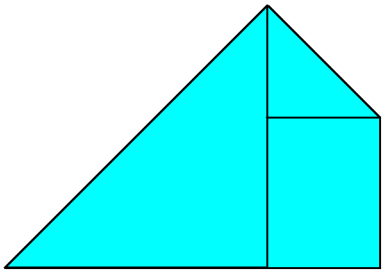
5. Fold the left hand edge onto the bottom edge.

6



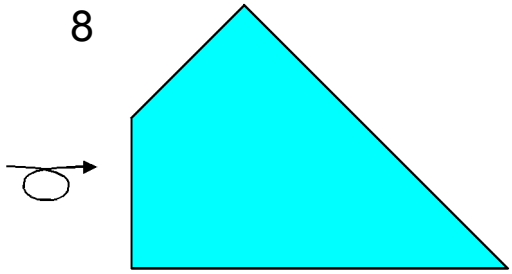
6. Fold the top edge onto the upright edge.

7



7. Turn over sideways.

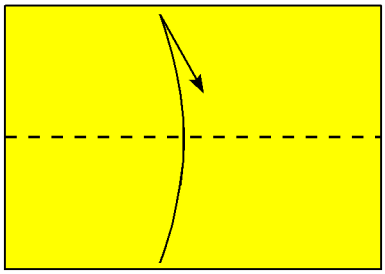
8



8. This is the finished tile. This method gives the largest tile of this shape that can be folded from a silver rectangle of any given size.

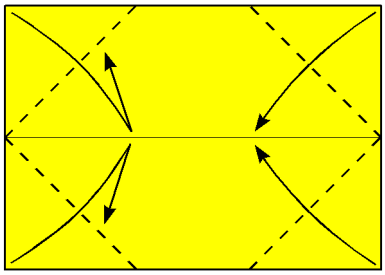
Folding from the silver rectangle - method 2

9



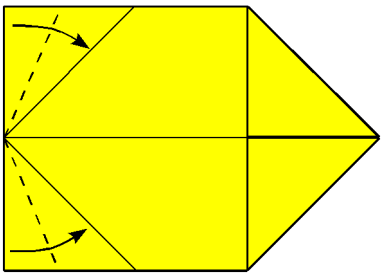
9. Fold in half downwards, then unfold.

10



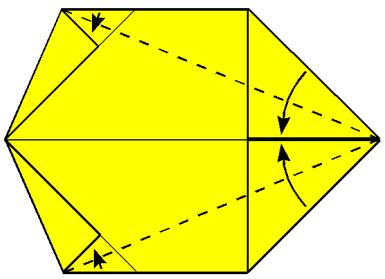
10. Fold all four corners inwards, using the horizontal crease to locate the folds. Unfold both folds on the left.

11



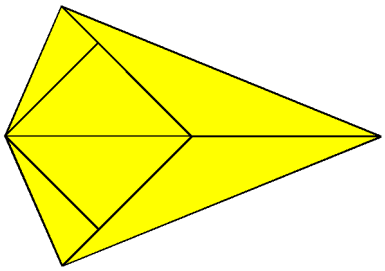
11. Fold both halves of the left edge inwards to lie along the sloping creases made in step 10.

12



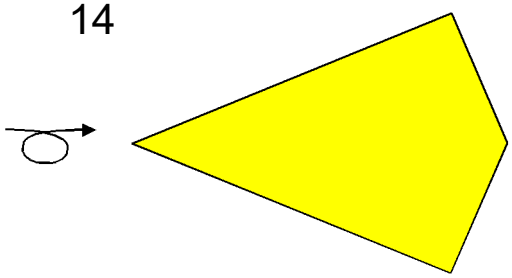
12. Fold the right top and bottom corners inwards as shown using the edges of the front flaps to locate the folds.

13



13. Turn over sideways.

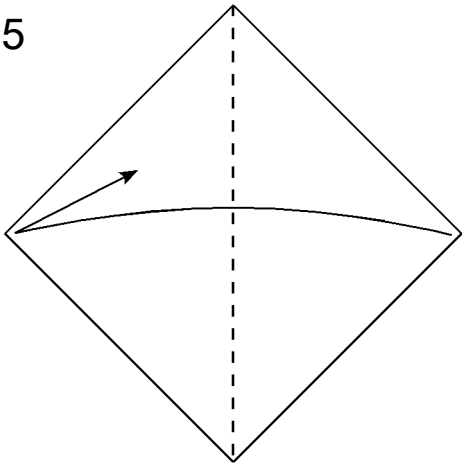
14



14. The octagon kite tile is finished.

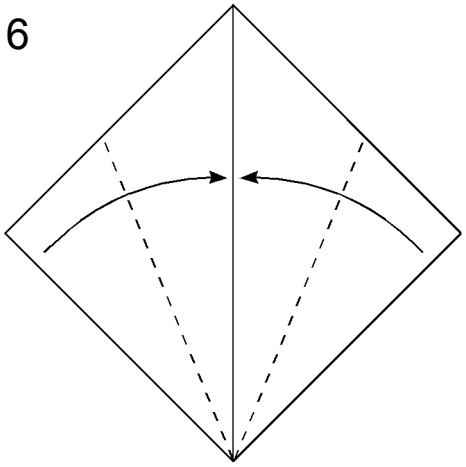
Folding from squares - method 1

15



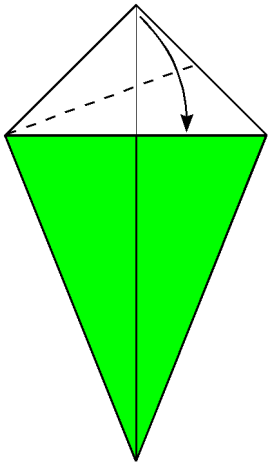
15. Fold in half sideways.

16



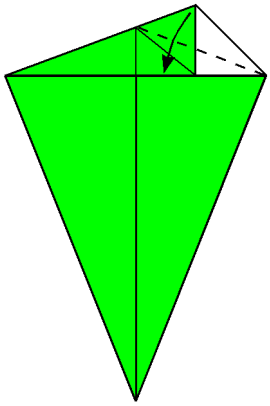
16. Fold both the left and right bottom sloping edges onto the vertical centre crease.

17



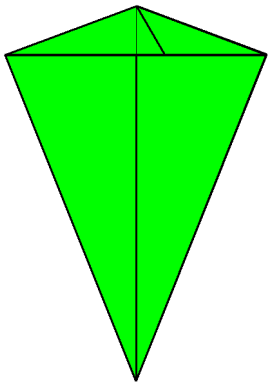
17. Fold the top left sloping edge downwards to lie along the top of the front layers.

18



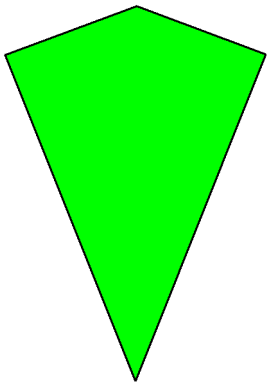
18. Fold the top right sloping edge downwards to lie along the top of the front layers.

19



19. Turn over sideways.

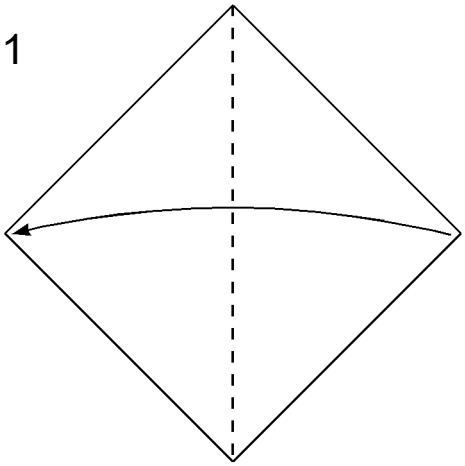
20



20. The octagon kite tile is finished.

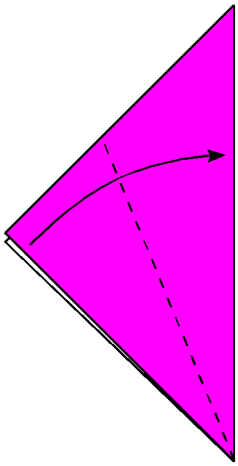
Folding from squares - method 2

21



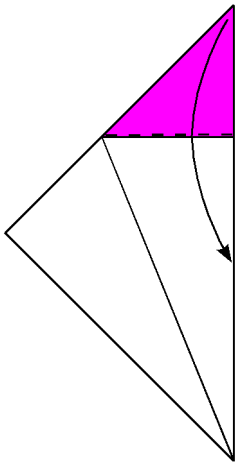
21. Fold in half sideways.

22



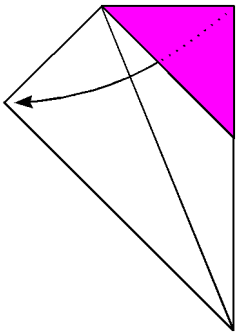
22. Fold the bottom left sloping edge of just the front layer onto the right hand edge.

23



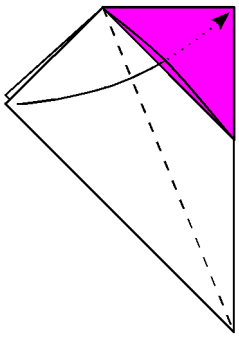
23. Fold the top point downwards along the line of the top edge of the front layers.

24



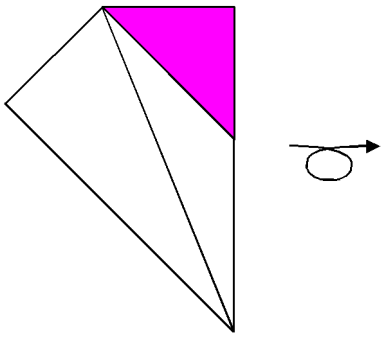
24. Open out the fold made in step 22.

25



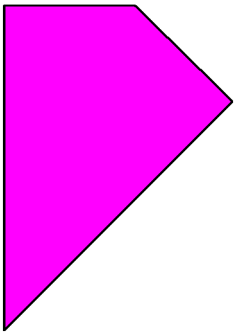
25. Remake fold 22 so that the left front corner tucks inside the pocket.

26



26. Turn over sideways.

27

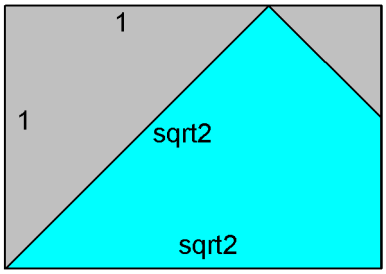


27. The octagon kite tile is finished.

Notes

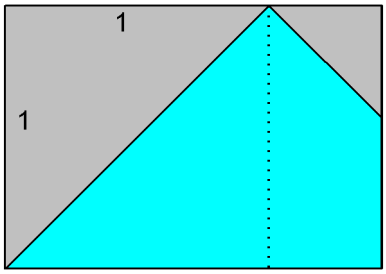
Calculating the perimeter of an octagon kite tile folded from a silver triangle using method 1.

A



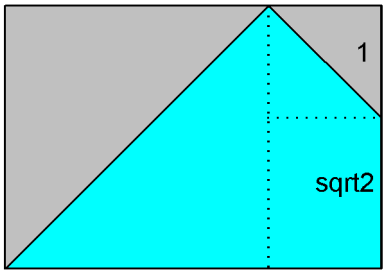
A. The length of the long side of a silver rectangle is sqrt2 a priori. Because the region above and to the left of the crease is a right angle isosceles triangle we know, by Pythagoras, that the length of its base is also sqrt2.

B



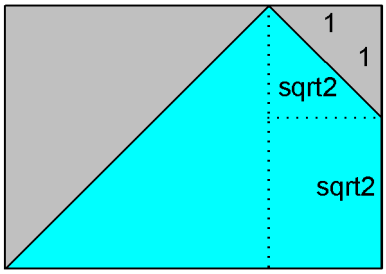
B. The equivalence of the short sides can be shown by remembering that a silver rectangle can be divided into a square and a leftover rectangle (see page xx) ...

C



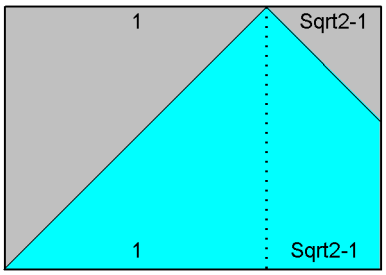
C. ... and that a leftover rectangle can in turn be divided into a square and a silver rectangle (see page xx).

D



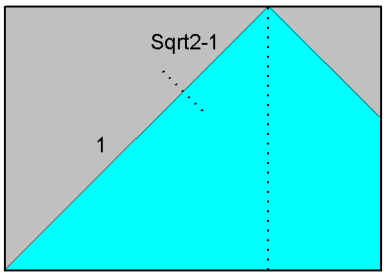
D. The upright short edge is sqrt2 a priori. The sloping short edge is sqrt2 by Pythagoras, since it forms the base of a right angle isosceles triangle whose other sides are of unit length.

E



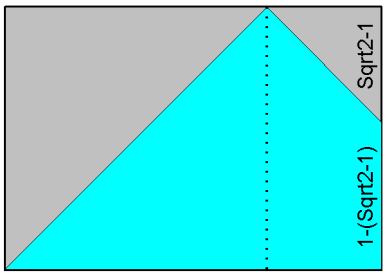
E. The long sides of a silver rectangle are sqrt2 a priori. They can be divided into lengths of 1 and Sqrt2-1 by a perpendicular line dropped from the top corner of the tile.

F



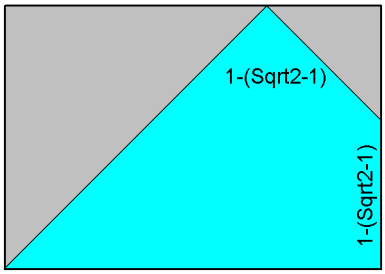
F. The diagonal long edge can be seen as having been divided in the same way. (It is not necessary to actually do this.)

G



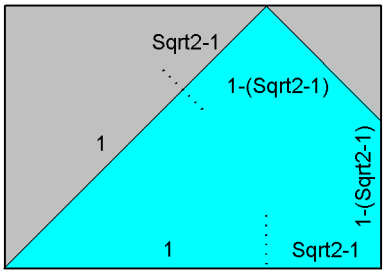
G. The right edge of the rectangle is divided into two at the apex of the tile. The length above the apex is Sqrt2-1 and that below it 1-(Sqrt2-1).

H



H. We know that the two short edges of the tile are of equal length.

I

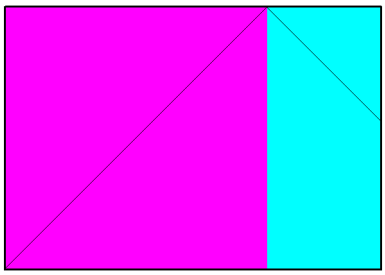


I. The edges of the tile are now divided into two sections of unit length and four smaller divisions, two short and two long. If we take one short and one long length from these remaining divisions we find that they add to unit length. 1- (Sqrt2-1) + Sqrt2-1 = 1. The perimeter of the Kite-shaped tile is therefore 4. A surprising result.

Other shapes with a perimeter of length 4

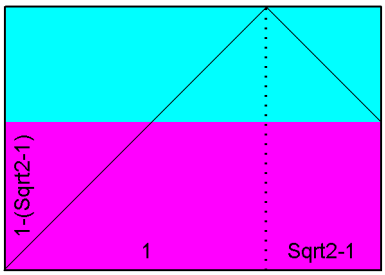
Two rectangles of perimeter 4 can be found using the corners of the octagon kite tile as location points.

J



J. This one is, of course, a square.

K

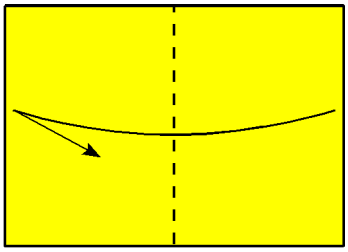


K. And this one is a leftover rectangle, in fact the largest such rectangle that can be obtained from a silver rectangle.

The silver rectangle can also be folded into two other tiles of perimeter 4.

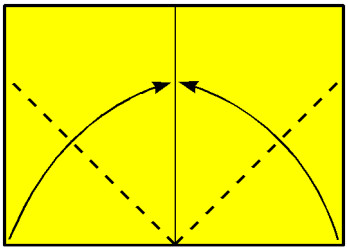
Folding the Double Trapezium Tile

L



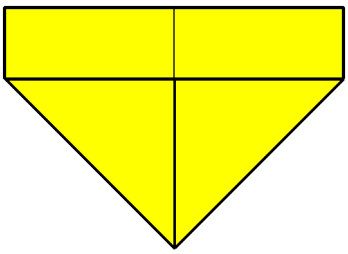
L. Fold in half sideways, then unfold.

M



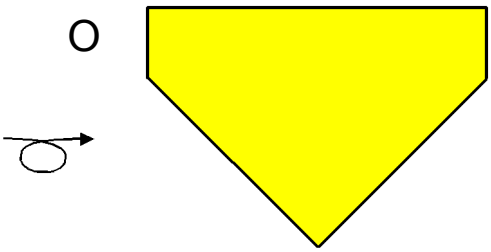
M. Fold both halves of the bottom edge onto the vertical crease.

N



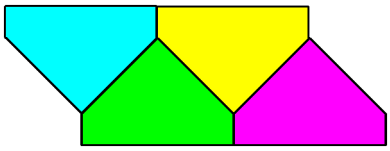
N. Make sure all the layers lie flat then turn over sideways.

O



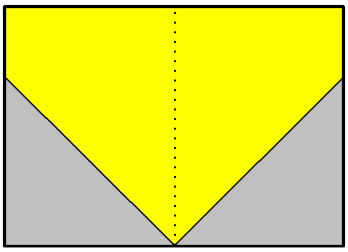
O. The double trapezium tile 1 is finished.

P



P. Double trapezium tiles will tile the plane like this.

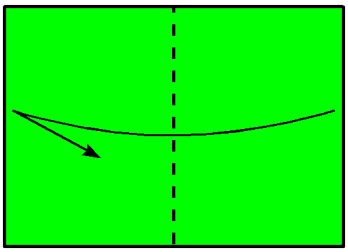
Q



Q. This tile is equivalent to two mirror-image trapezium tiles laid side by side. A trapezium tile has a perimeter of 3 in relation to the longest edge of the silver rectangle it is folded from (see separate pdf) Two tiles, of course, have a perimeter of 6, but two of their edges, both of unit length, are lost when joining them together, leaving a perimeter of 4.

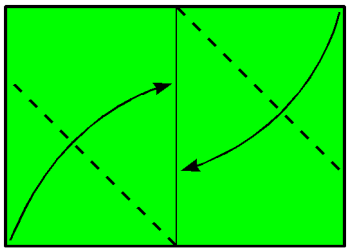
Folding the Rotated Double Trapezium Tile

R



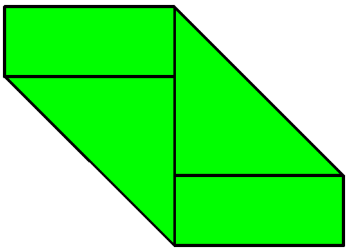
R. Fold in half sideways, then unfold.

S



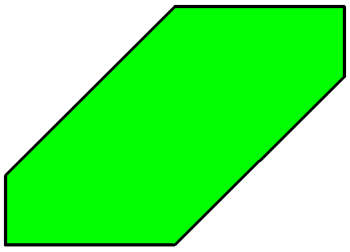
S. Fold the bottom left and top right corners inwards like this.

T



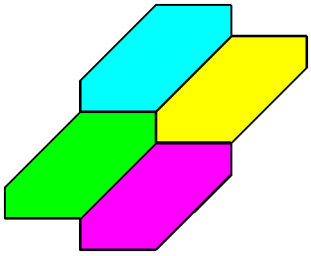
T. Check that all the layers lie flat then turn over sideways.

U



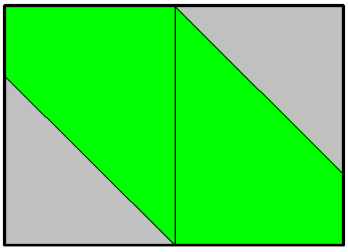
U. The Rotated Double Trapezium tile is finished.

V



V. Rotated double trapezium tiles will tile the plane like this.

W



W. This tile is also equivalent to two trapezium tiles laid side by side (one of which has first been rotated through 180 degrees). We know that a trapezium tile has a perimeter of 3 in relation to the longest edge of the silver rectangle it is folded from (see separate pdf). Two tiles, of course, have a perimeter of 6, but two of their edges, both of unit length, are lost when joining them together, leaving a perimeter of 4.