

Golden Proportion Tiles

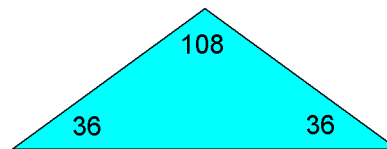
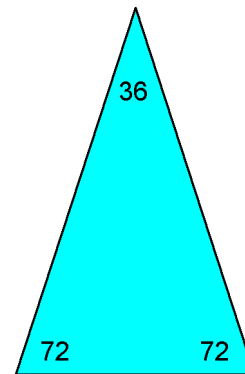
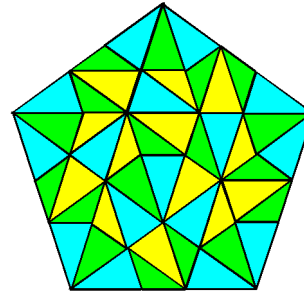
These diagrams show you how to fold the two Golden Proportion tiles shown below on the right from both silver rectangles and squares. Both tiles have edges which are in the golden proportion to one another. They will tile the plane individually or in combination in many different ways and will also go together to form compound tiles, of both shapes, and to form increasingly complex compound pentagons and pentagrams.

Because of their very different shapes, and because I like simple but descriptive names, I use the terms Mountain and Molehill to describe these tiles. You can probably guess which is which.

In order to keep the folding sequences simple the angles of these tiles have been approximated by using the natural geometry of the silver rectangle and mock platinum folding geometry derived from squares. Silver rectangles have edges in the proportion $1:\sqrt{2}$. DIN paper sizes such as A4, A5 etc are good enough approximations of silver rectangles for practical paperfolding purposes. A comparison of the angles obtained by these two different methods can be found on pages 16 and 17.

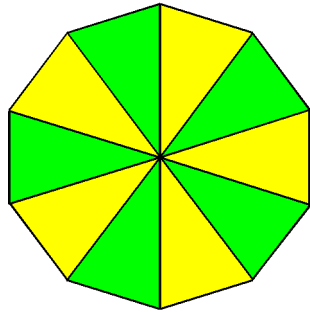
Compound tiles can also be made by using sets of tiles that have been scaled so that the long edges of one set of tiles are the same length as the short edges of the other.

I designed these tiles in 2000.



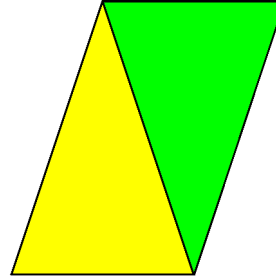
Tiling patterns

1



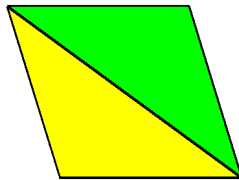
1. Ten Mountain tiles will fit together to form a decagon.

2



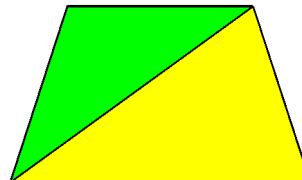
2. Two Mountain tiles will go together to form a parallelogram.

3



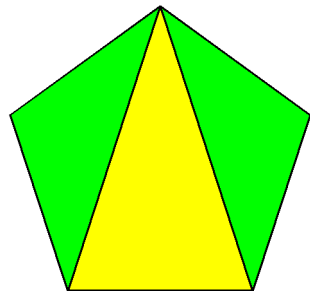
3. So will two Molehill tiles.

4



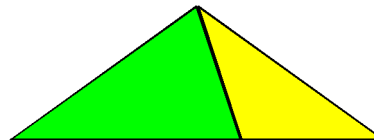
4. One Mountain and one Molehill will form a quadrilateral.

5



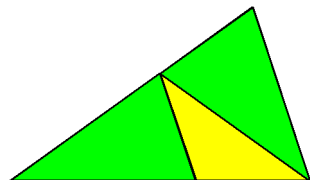
5. One Mountain tile and two Molehill tiles fit together to form a regular pentagon.

6



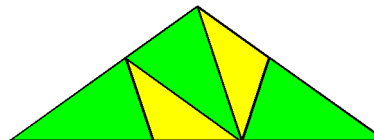
6. One Mountain and one Molehill tile will fit together to form a compound Molehill tile.

7



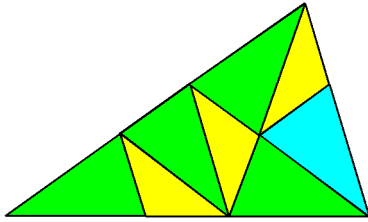
7. Two Mountain tiles and one Molehill tile will fit together to form a compound Mountain tile.

8

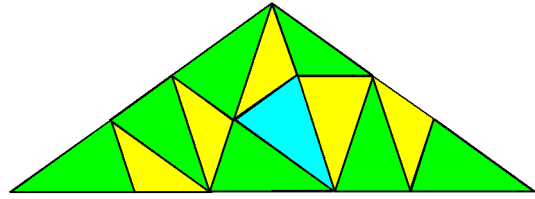


8. By combining these two compound tiles a larger compound Molehill tile is formed.

9

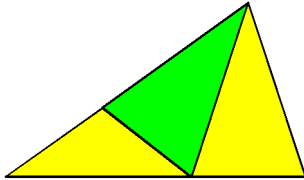


9. Each compound tile in this sequence can be combined with its predecessor to obtain a larger compound tile.



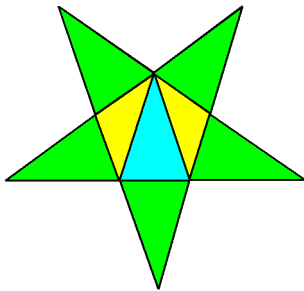
10. This process can be continued ad infinitum. The number of individual tiles in successive forms is given by the Fibonacci sequence.

11



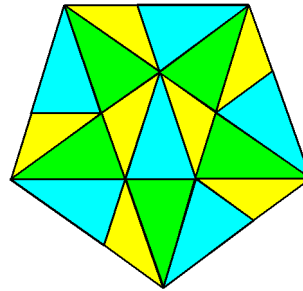
11. There is a second way of making a compound Mountain tile from two Mountain and one Molehill tile.

12



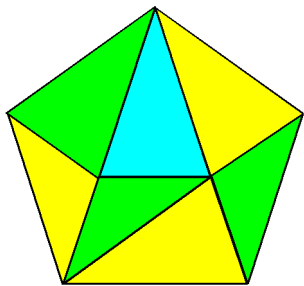
12. Mountain tiles can be added to the faces of the pentagon to create a pentagram.

13



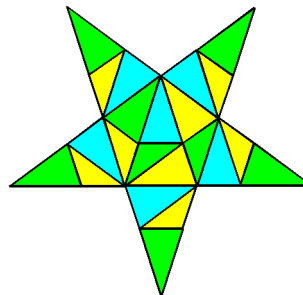
13. The pentagon can be filled to a more complex pentagon by adding compound Molehill tiles.

14



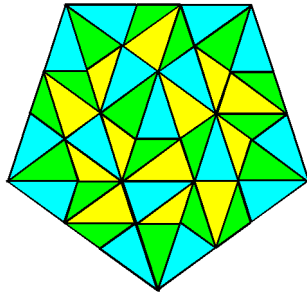
14. A compound pentagon can also be made like this by combining four Mountain and three Molehill tiles.

15



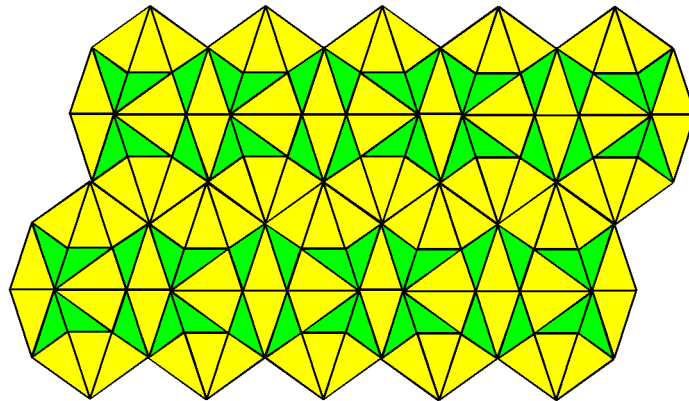
15. This pentagon can be turned into a pentagram by the addition of compound Mountain tiles.

16



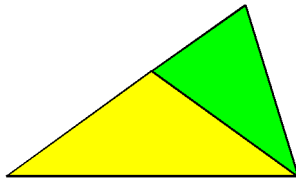
16. And filled to a more complicated compound pentagon still by adding compound Molehill tiles. This process can be continued ad infinitum.

17



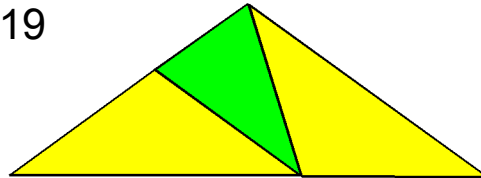
17. Mountain and Molehill tiles can also be arranged to tile the plane in other attractive ways. They are well worth

18



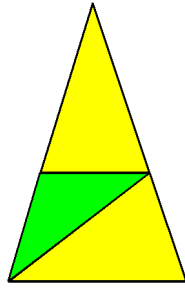
18. Golden proportion tiles can also be scaled so that one Molehill and one smaller Mountain tile will form a compound Mountain tile ...

19



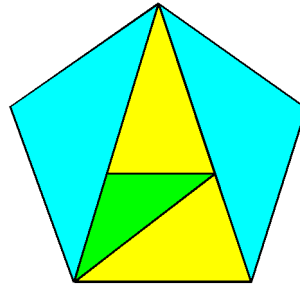
19. ... or two Molehill tiles and one smaller Mountain tile will form a compound Molehill tile like this.

20



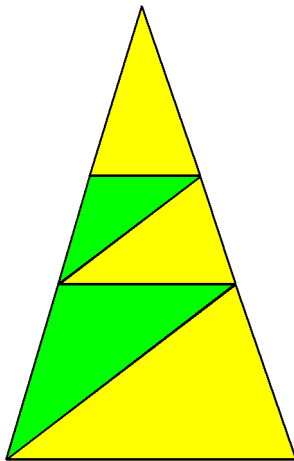
20. Two Mountain tiles can be also be combined with a smaller Molehill tile to form yet another compound Mountain tile ...

21



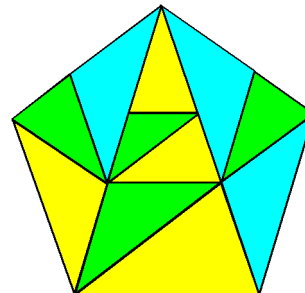
21. ... which can be filled to a pentagon with the addition of two large Molehill tiles.

22



22. Large and small Mountain and Molehill tiles can be used to create yet another compound Mountain tile. In theory this process can be extended ad infinitum.

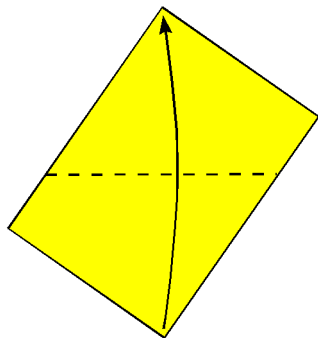
23



23. Many other compound pentagons can be created by combining tiles of different scales.

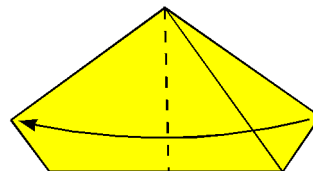
Folding the tiles from the silver rectangle The Mountain tile

24



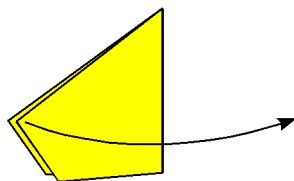
24. Fold in half corner to opposite corner.

25



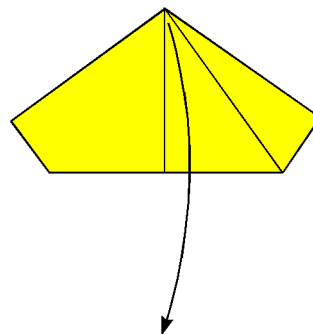
25. Fold in half sideways.

26



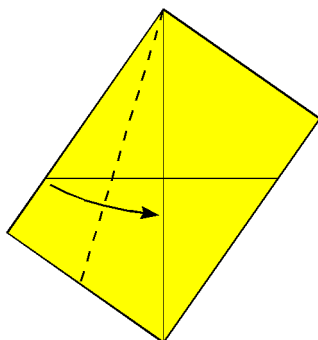
26. Open out the fold made in step 25.

27



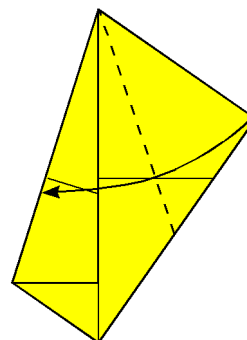
27. Open out the fold made in step 27.

28

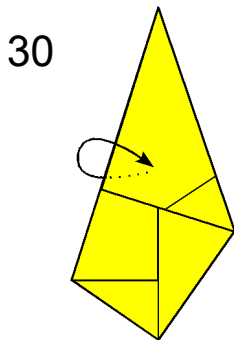


28. Fold the sloping left edge onto the crease made in step 27.

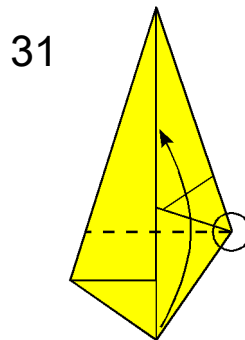
29



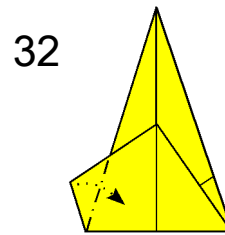
29. Fold the sloping top right edge onto the sloping top left edge.



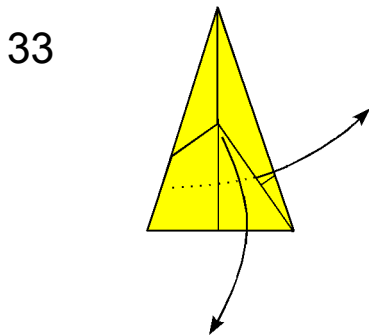
30. Bring the middle flap to the front.



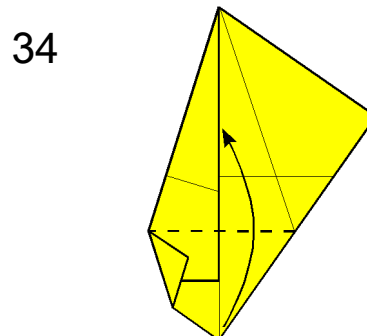
31. Fold the bottom point upwards using the right corner to locate the fold.



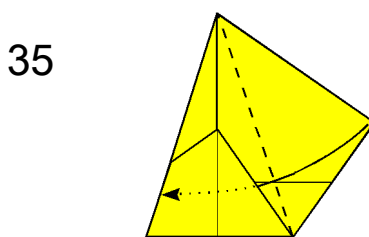
32. Fold the right hand corner behind out of sight.



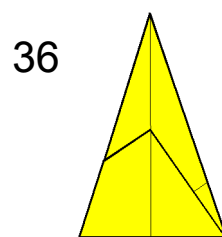
33. Open out the folds made in steps 30 and 31 (but not 32).



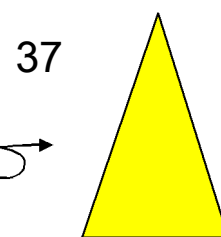
34. Fold the bottom corner upwards using the crease made in step 31.



35. Fold the right corner inwards, using the fold made in step 29, so that it goes behind the front layer and into the pocket immediately behind it.



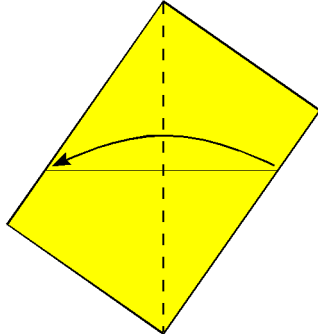
36. Make sure all the layers lie flat then turn over sideways.



37. The Mountain tile is finished.

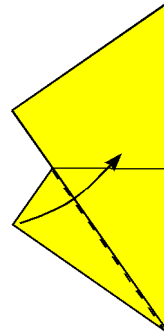
The Molehill tile

38



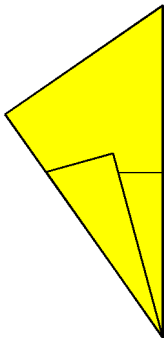
38. Begin by following steps 24 to 28 of the Mountain tile. Fold in half sideways using the existing crease.

39



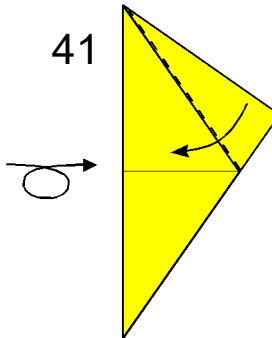
39. Fold the bottom left corner inwards around the edge of the front layer as far as it will go. Make sure the bottom point remains sharp.

40



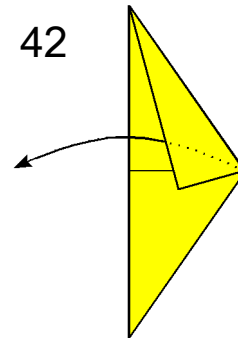
40. Turn over sideways.

41



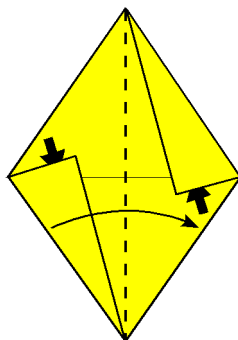
41. Repeat fold 39 on the top

42



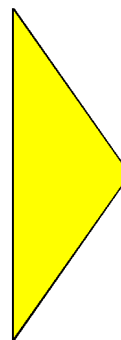
42. Open out the middle layers to the left.

43



43. Fold in half sideways, interlocking the two front flaps as you do so.

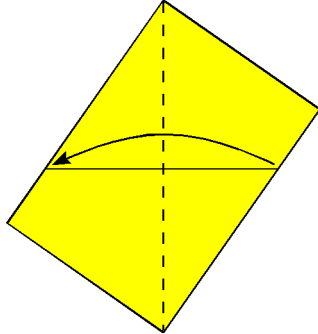
44



44. The Molehill tile is finished.

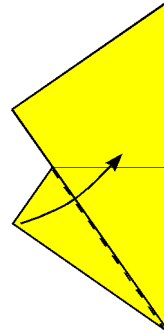
Scaling tiles folded from the silver rectangle

45



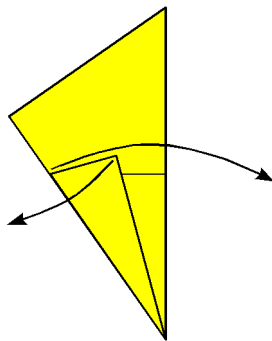
45. Begin by following steps 24 to 28 of the Mountain tile. Fold in half sideways using the existing crease.

46



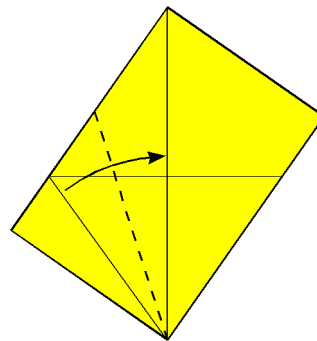
46. Fold the bottom left corner inwards around the edge of the front layer as far as it will go. Make sure the bottom point remains sharp.

47



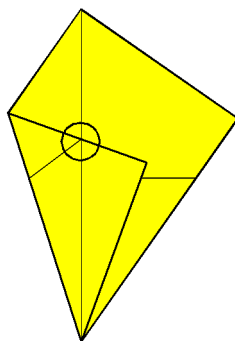
47. Open out fold 45 and 46.

48



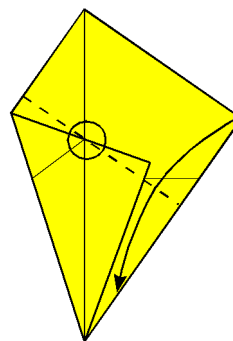
48. Fold the crease made in step 46 onto the diagonal crease.

49



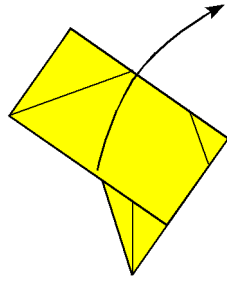
49. Check that the creases are aligned accurately like this and that the bottom point is sharp.

50



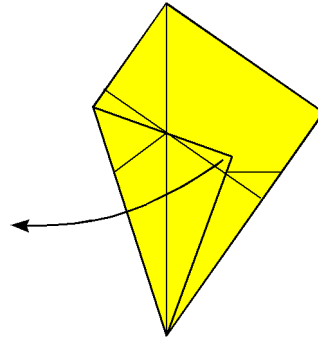
50. Fold the top part of the paper downwards so that the crease passes accurately through the point marked with a circle.

51



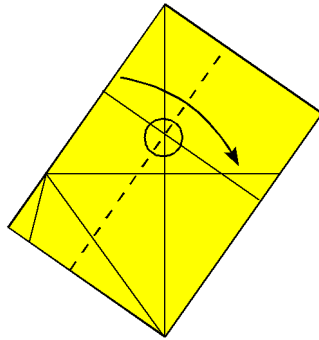
51. Make sure the edges of all the layers on the right hand side are accurately aligned. Open out the fold made in step 50.

52



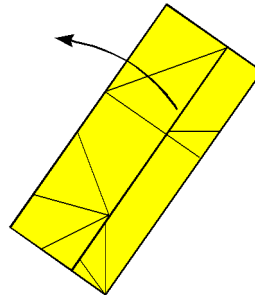
52. Open out the fold made in step 48.

53



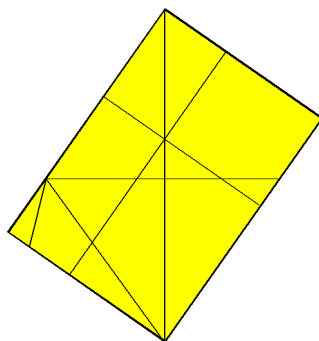
53. Fold the right hand part of the paper inwards so that the crease passes accurately through the point marked with a circle.

54



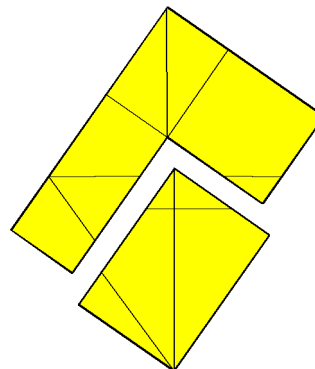
54. Make sure the edges of all the layers on the top and bottom edges are accurately aligned. Open out the fold made in step 53.

55



55. Cut along the creases made in steps 50 and 53 to remove the smaller silver rectangle.

56

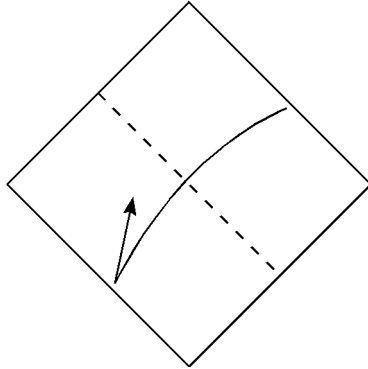


56. This smaller silver rectangle is the correct size to use to fold smaller scale tiles.

Folding the tiles from the square

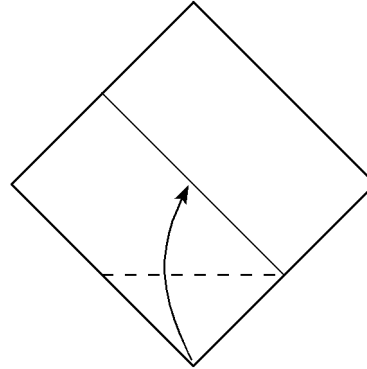
The Mountain tile

57



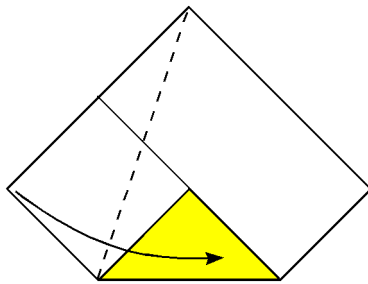
57. Fold in half edge to edge, then unfold.

58



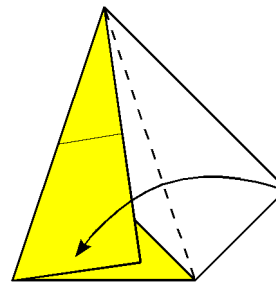
58. Fold the bottom corner to the centre.

59



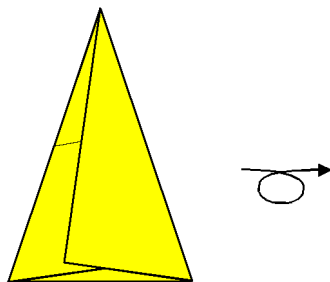
59. Fold the left hand corner inwards like this. Make sure the top point and the bottom left corner both remain sharp.

60



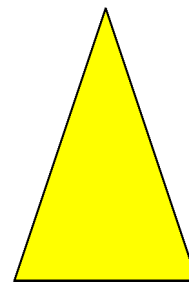
60. Repeat fold 58 on the right hand side of the design.

61



61. Make sure all the layers lie flat, then turn over.

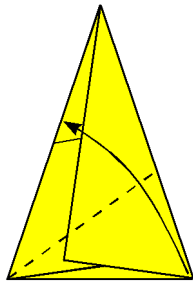
62



62. The Mountain tile is finished.

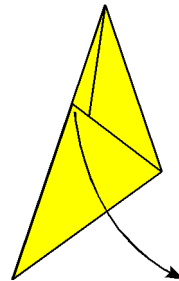
The Molehill tile

63



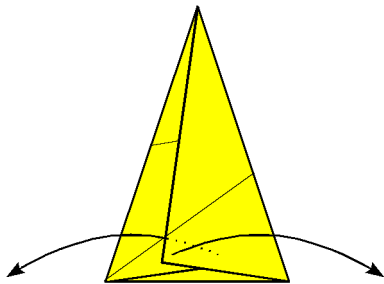
63. Begin with a finished Mountain tile. Fold the bottom edge onto the sloping left hand edge.

64



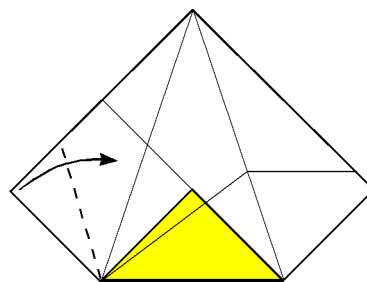
64. Open out the fold made in step 64.

65



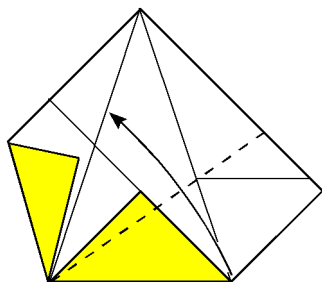
65. Open out the front two flaps.

66



66. Fold the left hand corner inwards like this.

67

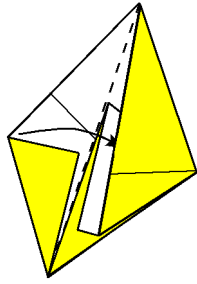


67. Fold the bottom right hand corner inwards, using, and extending, the crease made in step 63.

68

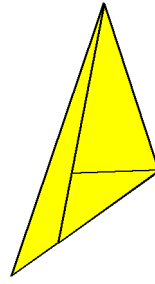
68. Make the fold shown using the existing crease then flatten the paper so that it looks like picture 69.

69



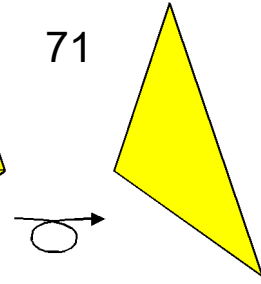
69. Fold the left hand flap inwards using the existing crease and tuck the left hand corner into the pocket underneath the front flap.

70



70. Make sure all the layers lie flat then turn over sideways.

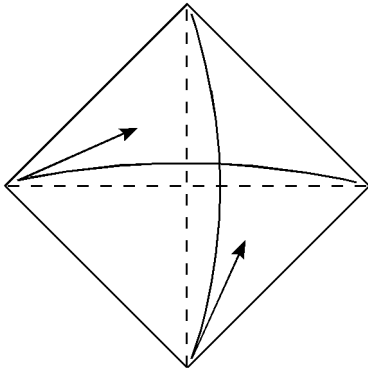
71



71. The Molehill tile is finished.

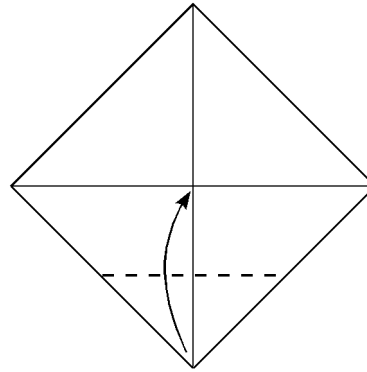
Scaling tiles folded from the square

72



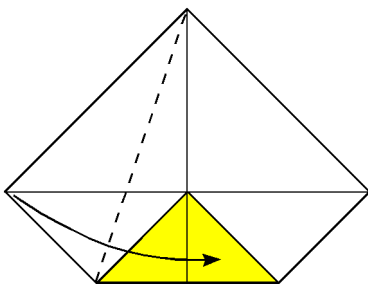
72. Fold in half corner to corner both ways, then unfold.

73



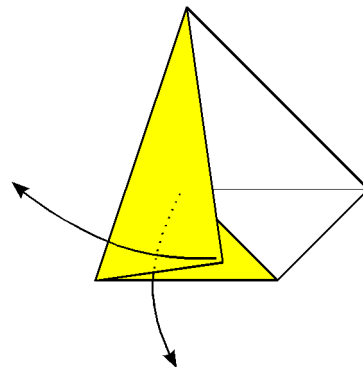
73. Fold the bottom corner to the centre.

74



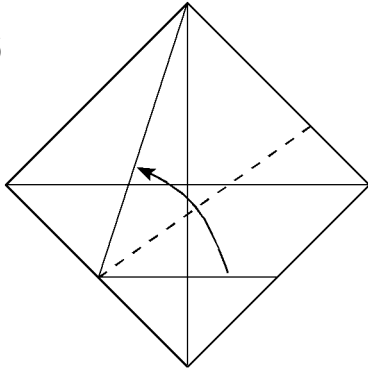
74. Fold the left hand corner inwards like this. Make sure the top point and the bottom left corner both remain sharp.

75



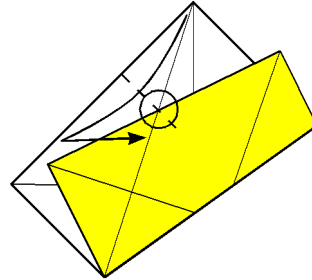
75. Open out the folds made in steps 76 and 74.

76



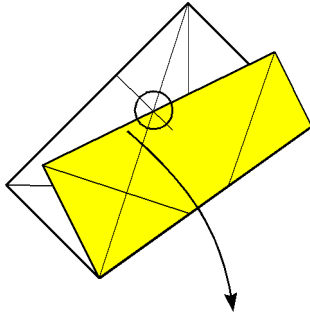
76. Fold the horizontal crease onto the sloping crease. Do this as accurately as possible.

77



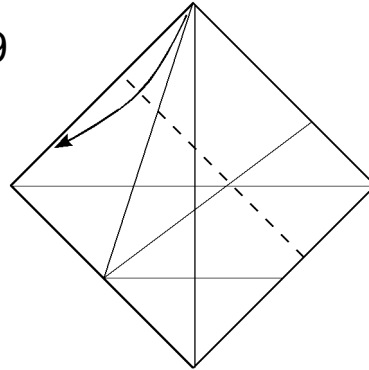
77. Fold the top corner onto the sloping top left edge to make a short crease that passes through the point marked with a circle. Do this as accurately as possible.

78



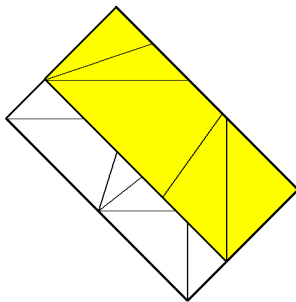
78. Open out the fold made in step 76.

79



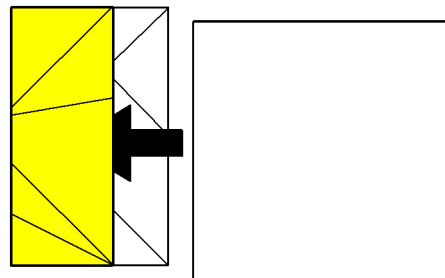
79. Extend the fold made in step 77 across the whole width of the paper.

80



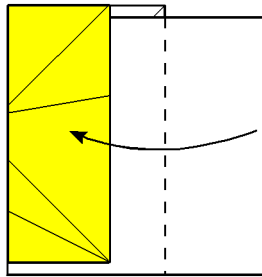
80. Make sure the edges of both layers along the top left and bottom right edges are aligned. This is the template.

81



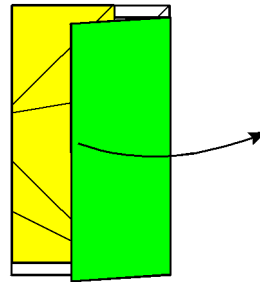
81. Slide a second square of the same size into the template like this.

82



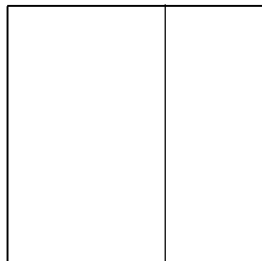
82. Fold the right edge inwards so that it lines up with the right edge of the template.

83



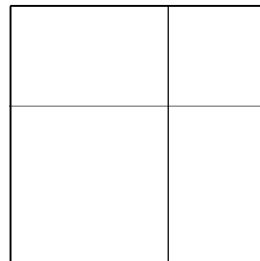
83. Open out and remove from the template.

84



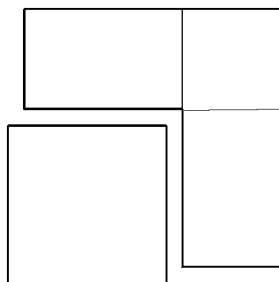
84. By rotating the paper through ninety degrees and repeating steps 81 to 83 ...

85



85. ... you will produce a crease pattern like this.

86

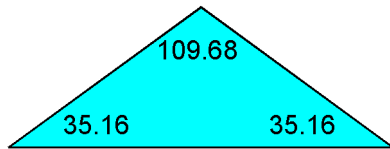


86. Cut out the smaller square like this. This square is the correct size to fold smaller scale tiles from (see pages 4 and 5).

Comparing approximated angles

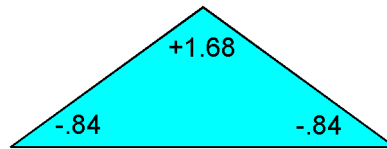
Tiles folded from the silver rectangle

87



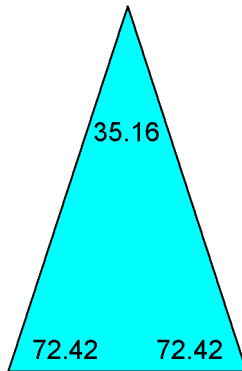
87. These are the approximated angles of the molehill tile

88



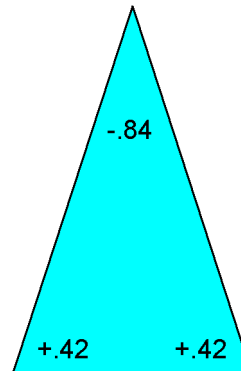
88. The errors are distributed like this.

89



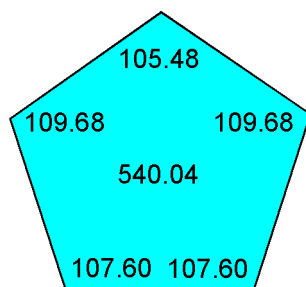
89. These are the approximated angles of the mountain tile.

90



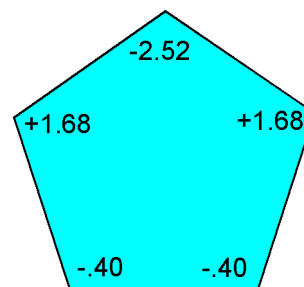
90. The errors are distributed like this.

91



91. When two molehill tiles and one mountain tile are combined into a pentagon the angles of each corner look like this.

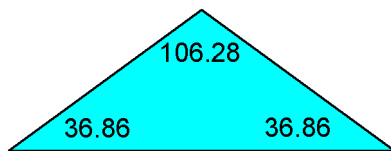
92



92. And the errors will be distributed like this.

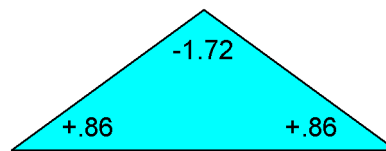
Tiles folded from the square

93



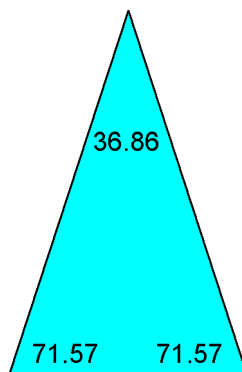
93. These are the approximated angles of the molehill tile.

94



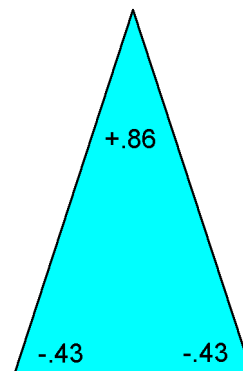
94. The errors are distributed like this.

95



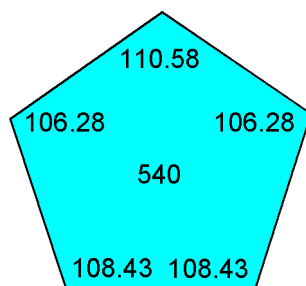
95. These are the approximated angles of the mountain tile.

96



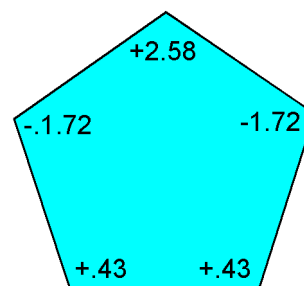
96. The errors are distributed like this.

97



97. When two molehill tiles and one mountain tile are combined into a pentagon the angles of each corner look like this.

98



98. And the errors will be distributed like this.

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