

# Cairo Tiles

Designed by David Mitchell

Cairo tiles are irregular pentagons. The internal angles of the tile are shown in the picture to the right below.

These diagrams show you how to fold Cairo tiles from squares, silver, or  $1:\sqrt{2}$ , rectangles, leftover, or  $1:\sqrt{2}+1$ , rectangles and US letter size paper, and how to use them to construct the Cairo tessellation. A4 paper is a good approximation of a silver rectangle. The leftover rectangle is the piece left over if you remove the largest possible square from a silver rectangle.

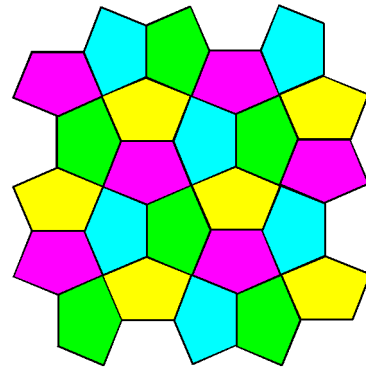
The simplest way to arrive at the Cairo tile is to fold a leftover rectangle in half diagonally from corner to opposite corner.

If you divide a silver rectangle into a square and a leftover rectangle and fold a Cairo tile from both shapes using the methods given on pages 4 and 5 you will find that the two tiles are of exactly the same size.

Both A4 and US letter size paper can easily be folded down to the dimensions of a leftover rectangle. One way of doing this is given on pages 10 and 11. There are several ways to fold a Cairo tile directly from a silver rectangle.

I have also included a way of folding a decorative version of the Cairo tile from a square which builds into a visually stunning multicoloured tessellation.

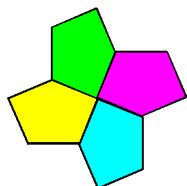
I first discovered how to fold a Cairo tile from a leftover rectangle in 2000. The methods from a square and silver rectangles were discovered shortly thereafter. I designed the decorative version of the tile in 2003.



## Constructing the Cairo tessellation

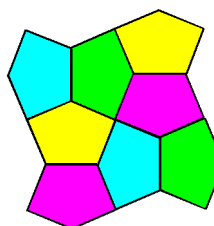
There are several interesting ways to construct a Cairo tessellation. If tiles of four colours are used all the tiles of the same colour can be used in the same orientation throughout the pattern.

1



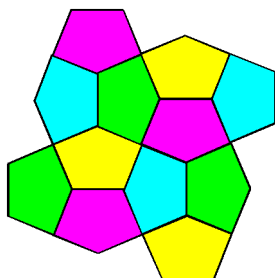
1. Begin by arranging four tiles in a distorted cross pattern like this.

2



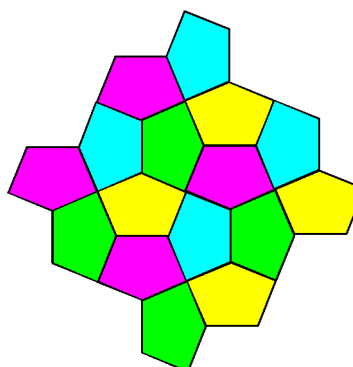
2. Add four more tiles making sure the orientation of each colour is maintained.

3



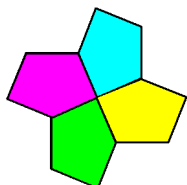
3. Continue adding tiles to each edge of the tessellation, always making sure that the orientation of each colour is maintained.

4



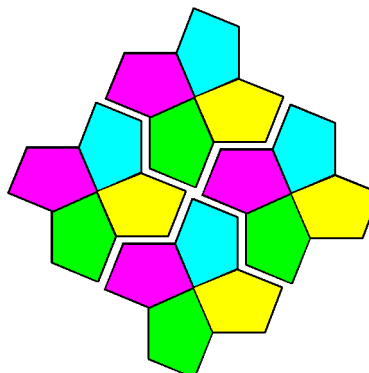
4. The pattern will continue to grow outwards like this.

5

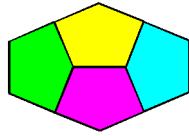


5 and 6 . Alternatively, you can see this pattern as a number of distorted crosses tiled together.

6

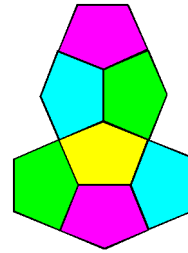


7



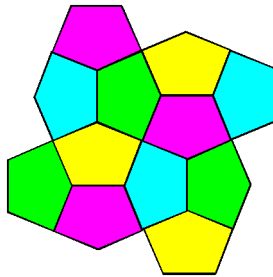
7. Alternatively, the pattern shown in picture 3 can be achieved by first arranging four tiles in the form of an elongated hexagon like this ...

8



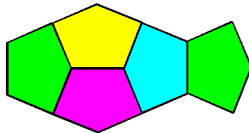
8. ... then adding three more tiles to form a second elongated hexagon at right angles to the first. The yellow tile is shared between both hexagons.

9



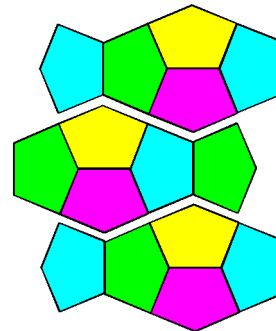
9. Add six more tiles to create four intersecting hexagons like this. This pattern is now the same as picture 3.

10



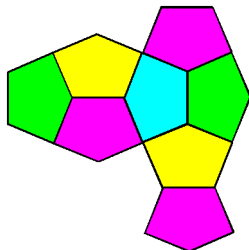
10. On one occasion I was building this tessellation with a class of 7 and 8 year old when one of them pointed out to me that you could also construct the tessellation using fish.

11



11. Fish that swim in different directions can be tiled together.

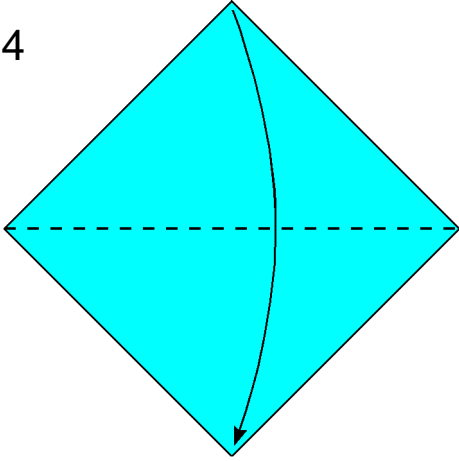
12



12. The tessellation can also be built from fish that swim at right angles to each other.

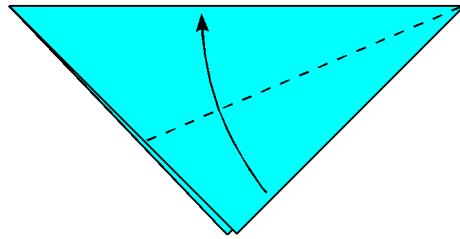
## Folding the Cairo tile from a square

14



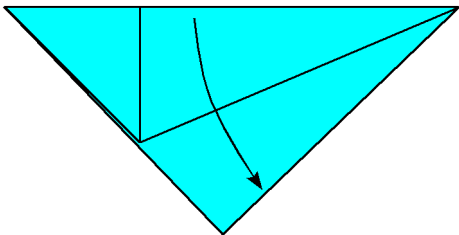
14. Arrange your square like this and fold it in half downwards.

15



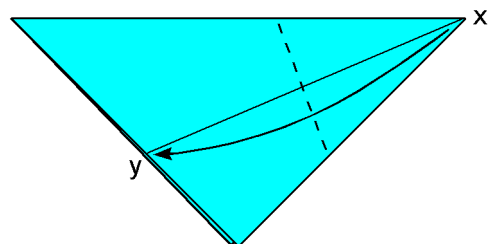
15. Fold the sloping right bottom edge of the front layer upwards so that it lies along the top edge.

16



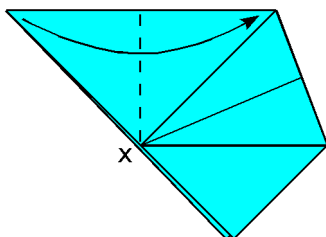
16. Open out the fold made in step 15.

17



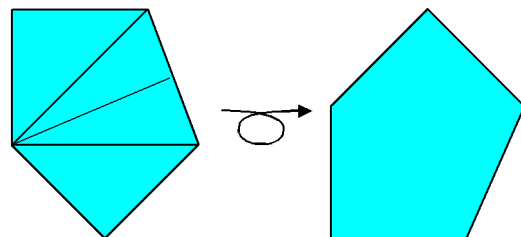
17. Fold corner x onto y.

18



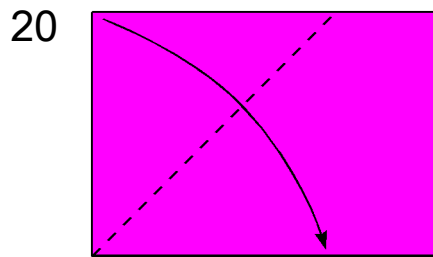
18. Fold the top left corner inwards as shown.

19

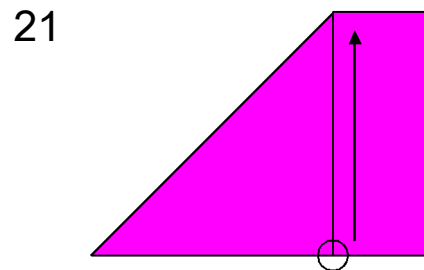


19. Flatten the creases and turn over. The Cairo tile is achieved.

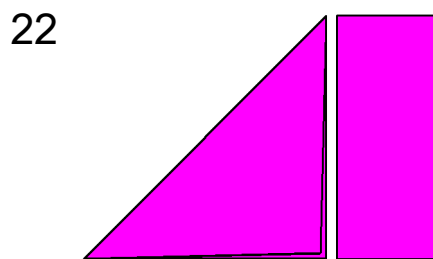
## Folding the Cairo tile from a leftover rectangle



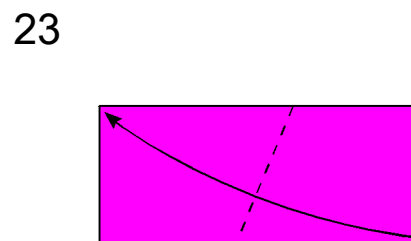
20. To obtain a leftover rectangle from a silver rectangle (such as a sheet of A4 sized paper) first fold the left edge onto the bottom edge like this.



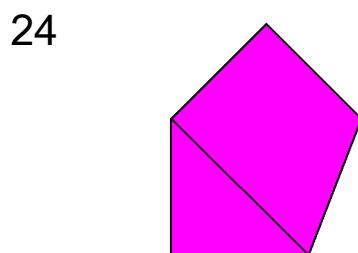
21. Cut upwards along the right edge of the top layer to separate the two pieces.



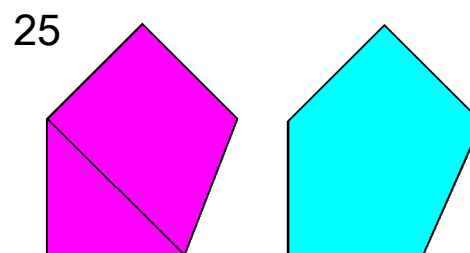
22. The right hand piece is a leftover rectangle. The left hand piece can be unfolded to a square. Set this piece aside.



23. Fold the leftover rectangle in half corner to opposite corner like this.



24. The Cairo tile is achieved.

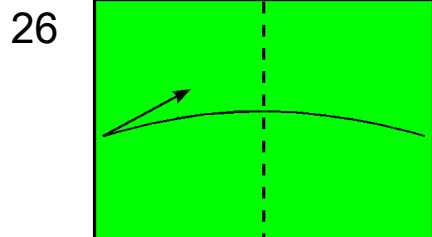


25. If you were to fold the square we set aside at step 22 into another Cairo tile using the method given in steps 14 to 19 you would find that it was of exactly the same size.

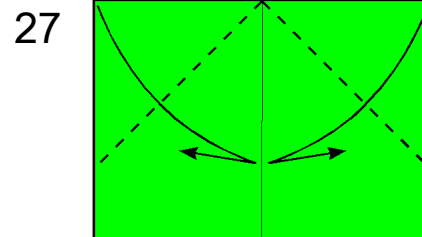
## Folding the Cairo tile from a silver rectangle

### Method 1

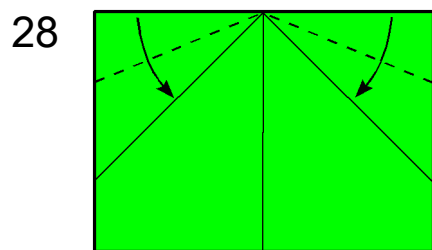
This method produces the largest Cairo tile that can be folded from a silver rectangle.



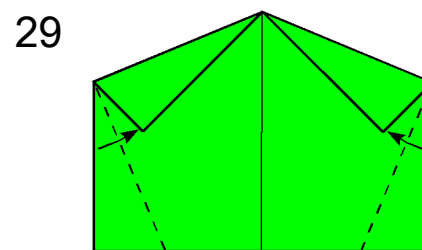
26. Fold in half sideways, then unfold.



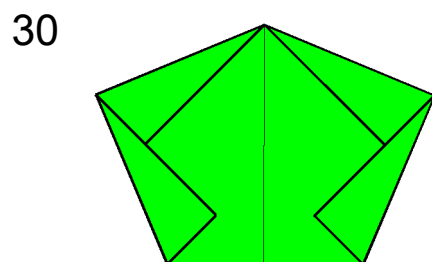
27. Fold both top corners inwards using the crease made in step 26 to locate the folds, then unfold.



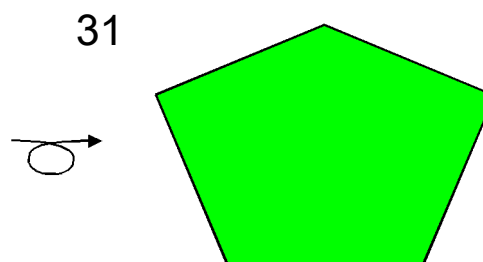
28. Fold both halves of the top edge onto the creases made in step 28.



29. Fold both the right and left edges inwards using the edges of the front flaps to locate the folds.

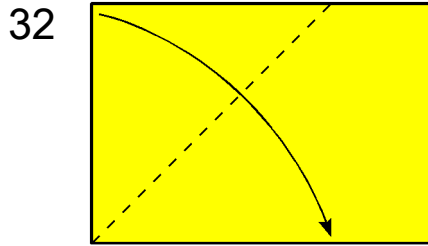


30. Turn over.

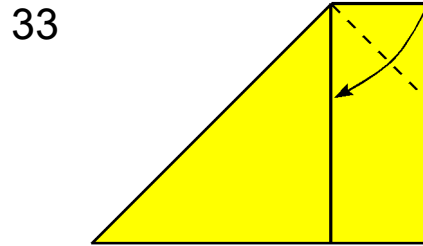


31. The Cairo tile is achieved.

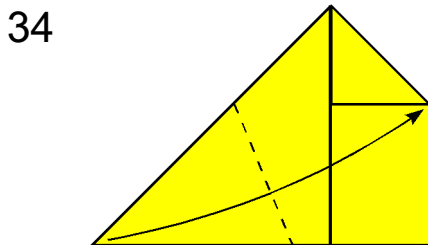
## Method 2



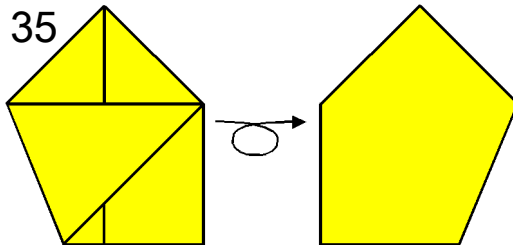
32. Begin by folding the left edge onto the bottom edge.



33. Fold the top edge to butt against the vertical right edge of the front layer.

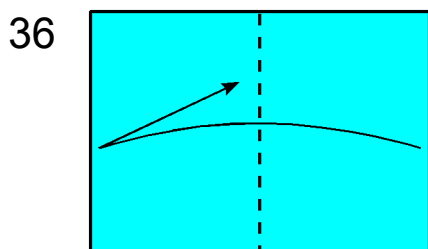


34. Fold the left point onto the top of the right edge.

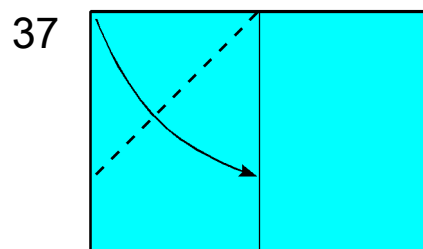


35. Turn over. The Cairo tile is achieved.

## Method 3

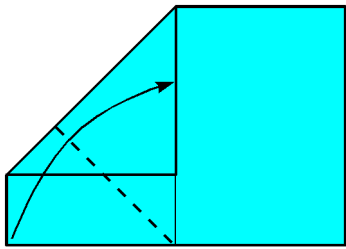


36. Fold in half sideways, then unfold.



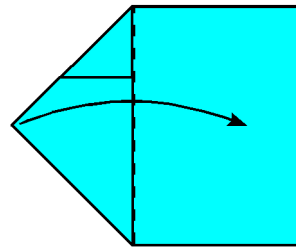
37. Fold the left hand half of the top edge onto the vertical crease.

38



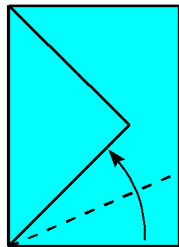
38. Fold the left hand half of the bottom edge onto the vertical crease.

39



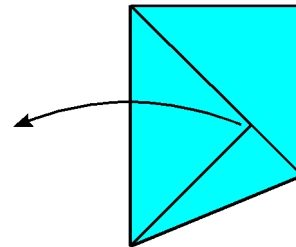
39. Fold the left corner across to the right using the existing vertical crease.

40



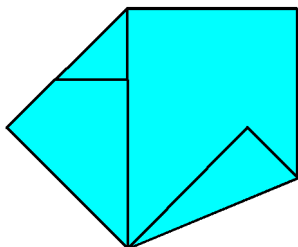
40. Fold the right hand half of the bottom edge inwards to butt against the bottom right sloping edge of the front layers.

41



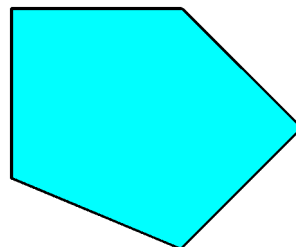
41. Open out the fold made in step 39.

42



42. Turn over.

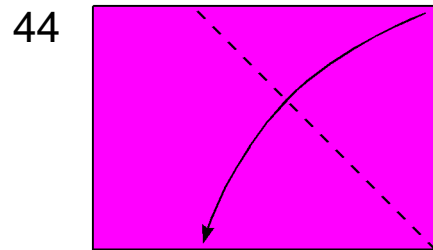
43



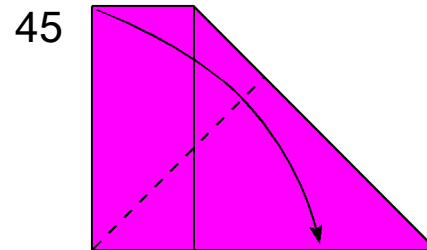
43. The Cairo tile is achieved.



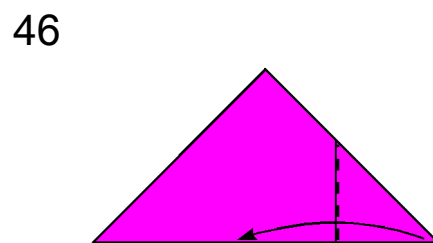
## Method 4



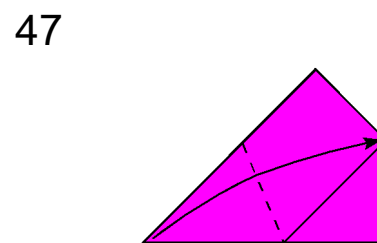
44. Fold the right edge onto the bottom edge.



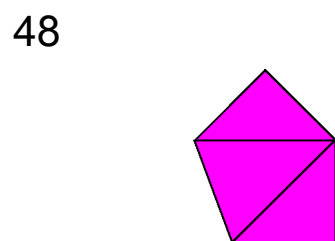
45. Fold the left edge onto the bottom edge.



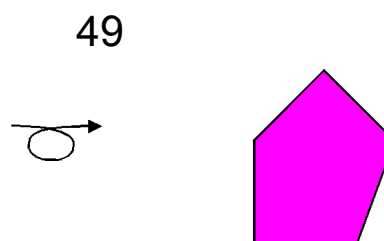
46. Fold the right point inwards along the line of the right edge of the front layers.



47. Fold the left point onto the top of the right edge.



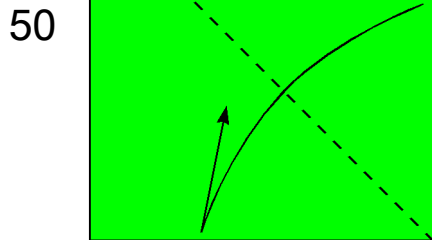
48. Turn over.



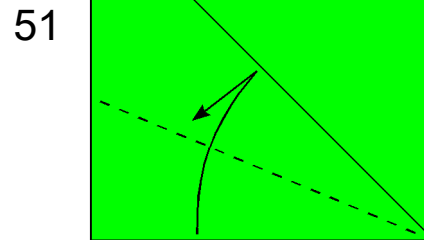
49. The Cairo tile is achieved.

## Folding leftover rectangles from US letter size paper

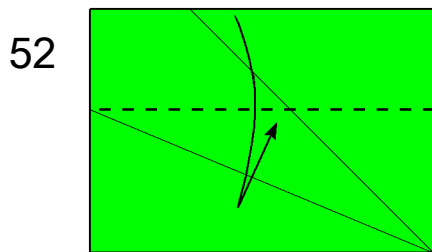
This method can also be used to fold leftover rectangles from A4.



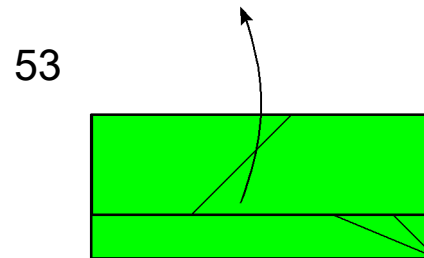
50. Fold the right edge onto the bottom edge, then unfold.



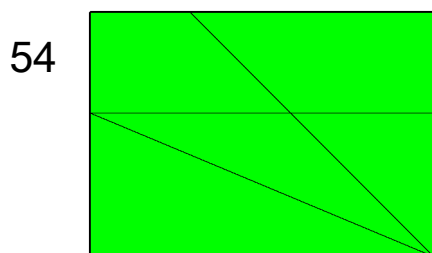
51. Fold the bottom edge onto the crease made in step 50, then unfold.



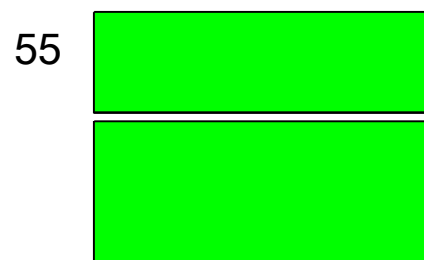
52. Fold the top edge downwards, using the point where the crease made in step 51 intersects the left edge to locate the fold, and making sure that the right and left edges are aligned to ensure the fold is horizontal.



53. Unfold.

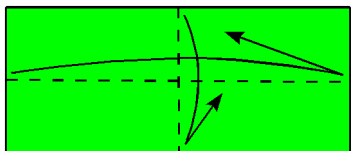


54. Separate the paper into two parts by cutting along the horizontal crease.



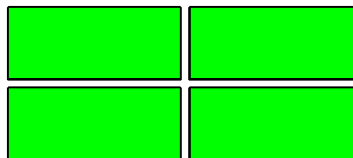
55. The lower rectangle is a leftover rectangle. The upper rectangle can be discarded.

56



56. It is a simple matter to divide this large leftover rectangle into four smaller ones by folding it in half edge to edge both ways and cutting along the resulting creases.

57

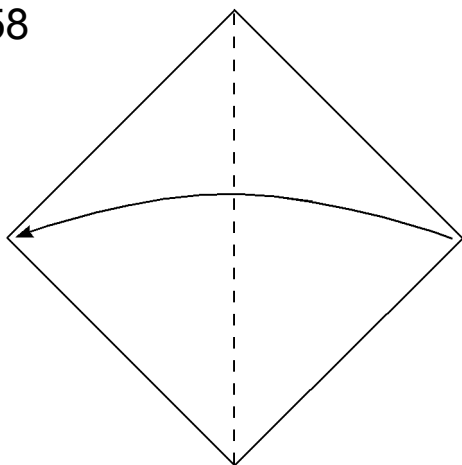


57. These leftover rectangles can be turned into Cairo tiles in the way shown on page 5.

### Folding a decorative Cairo tile from a square

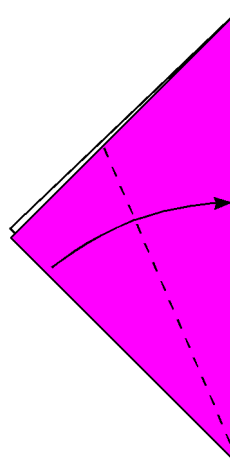
You will need a small square of irogami for each tile. Begin with your paper arranged white side up.

58



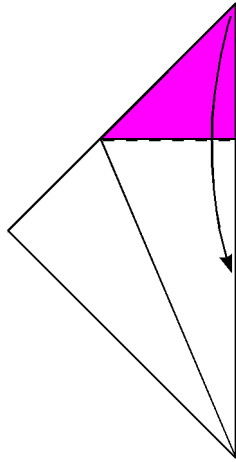
58. Fold in half corner to corner.

59



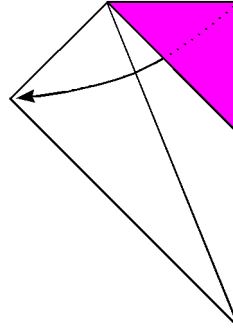
59. Fold the bottom left sloping edge of the front layer onto the right hand edge.

60



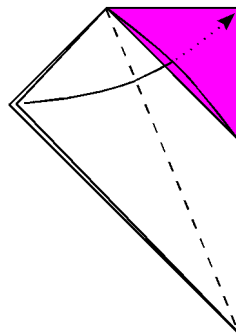
60. Fold the top point downwards like this.

61



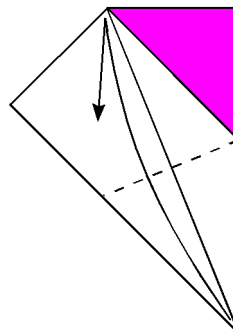
61. Undo the fold made in step 59.

62



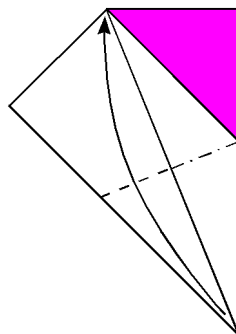
62. Remake the fold but tuck the corner into the pocket between the two front layers.

63



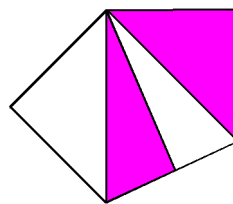
63. Fold the tile in half upwards like this then unfold.

64



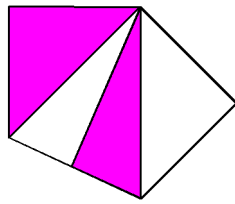
64. Remake fold 63 in between the other layers.

65



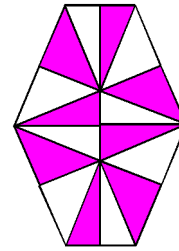
65. This is the result. Make sure all the layers lie flat.

66



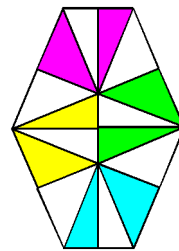
66. The tiling patterns shown in steps 67 to 79 use a combination of equal numbers of tiles and mirror image pattern tiles. Mirror image tiles are easily folded by making all the folds in mirror image. Tiles of just one type will fit together to make patterns but some areas of the same colour will stretch across the boundaries between the tiles.

67



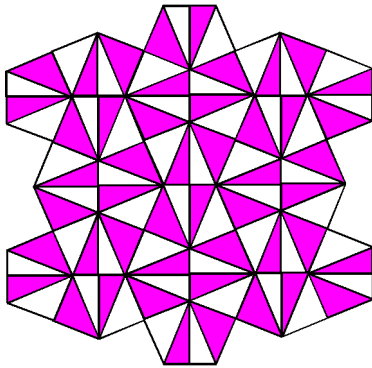
67. Four tiles, two of each type, folded from the same colour paper will form a hexagon that looks like this.

68



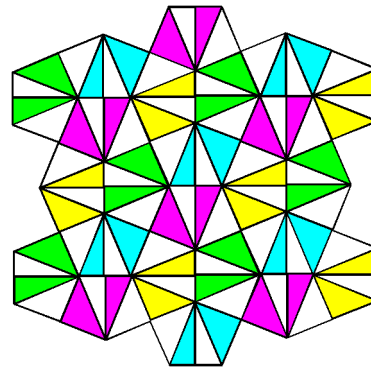
68. Four tiles, two of each type, folded from different coloured papers will form a hexagon that looks like this.

69



69. A 'turtle' of single colour tiles, using equal numbers of each type, looks like this ...

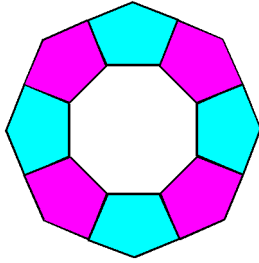
70



70. ... and one made by maintaining the orientation of tiles of four different colours like this.

And finally ...

71



71. It is perhaps also worth noting that eight Cairo tiles will tile together to form a hollow octagonal ring like this.

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