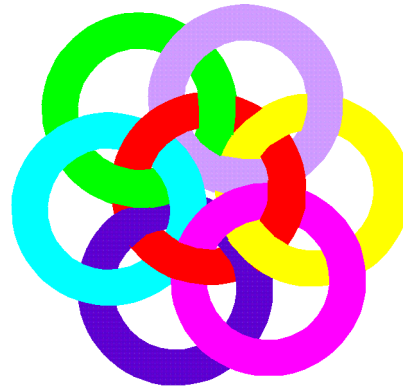


# Borromean Rings, Bands, Figures of Eight and Trefoils

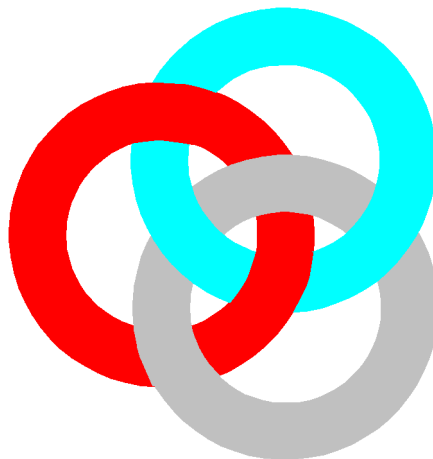
This material was originally written in 1999 as part of a proposal for a book to be called *Implausible Objects* which I was writing for Tarquin publications. Unfortunately the book never made it to print.



The material on how to transform the Borromean rings into their alternate forms is adapted from a published routine for a conjuring trick by Robert E Neale. The Borromean Cube with holes is from an unpublished MS by Robert E Neale. All other material is my own.

Seven rings linked, or rather not linked, in a Borromean relationship. This arrangement can be extended ad infinitum.

## Borromean Rings

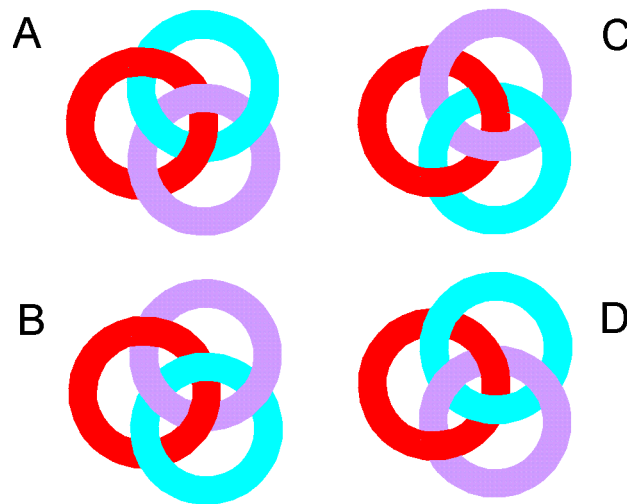


The Borromean Rings are an implausible object because despite the fact that none of the rings is linked to either of the others they cannot be separated without cutting one of them apart.

To see this look at the rings in pairs. The red and blue rings aren't linked. Nor are the blue and purple rings, or the purple and red rings. No two rings are linked. But it's impossible to remove any of them from the design.

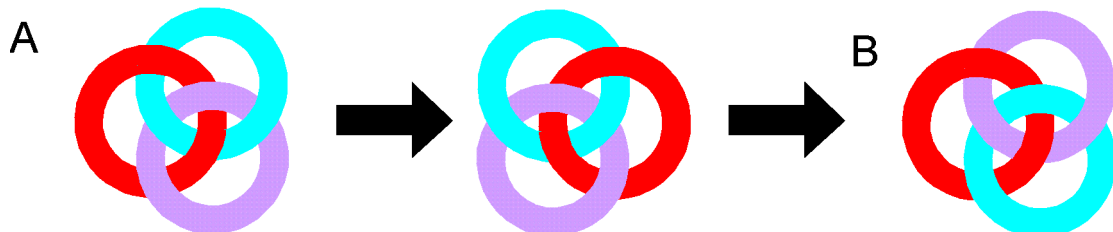
Another way of looking at it is to realise that the red ring is on top of the blue ring which is on top of the purple ring which is on top of the red ring.

Provided each ring is coloured the same both sides. there are just four different ways to arrange three rings like this.



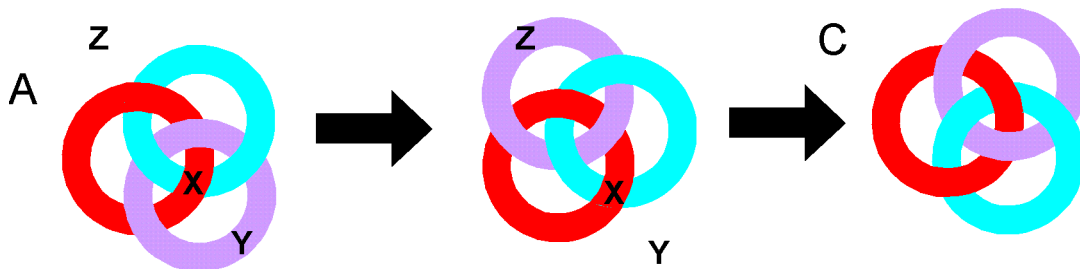
Oddly, although you cannot take the rings apart you can change them round from one arrangement to another while they are linked, or rather not linked, together.

The first transformation is easy. By turning arrangement A over and then swivelling it around you can get to arrangement B.



Turning arrangement B over and swivelling it around will bring you back to A.

Finding arrangement C is only a little harder. Begin with arrangement A. With your right hand hold the blue and red rings between finger and thumb at point X. Your thumb has to go underneath the purple ring to do this. Then with your left hand take hold of the purple ring at point Y and pull it towards Z. As you do this two things will happen. The purple ring will be turned over forwards and while doing so it will pass entirely outside the blue ring. Flatten the arrangement down, swivel it around and you end up with arrangement C.

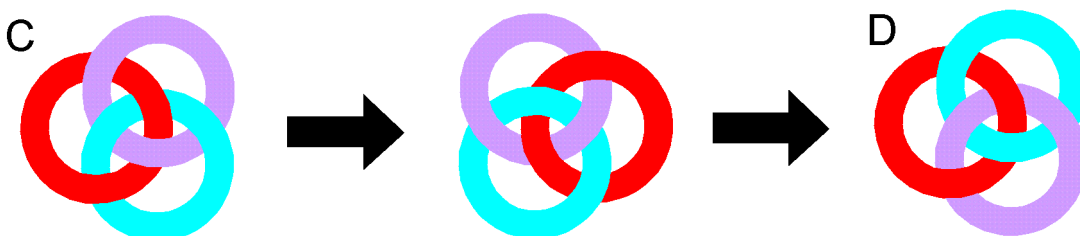


To return to arrangement A simply reverse the procedure, again turning the purple ring over forwards while passing it entirely outside the blue ring.

Now try holding the blue and red rings together at point X with your right finger and thumb above the purple ring, then reaching behind the arrangement with your left hand to pick up the purple ring at point Y. This time turn the purple ring over backwards and pass it completely outside the red ring. Surprisingly, the result is the same - arrangement C.

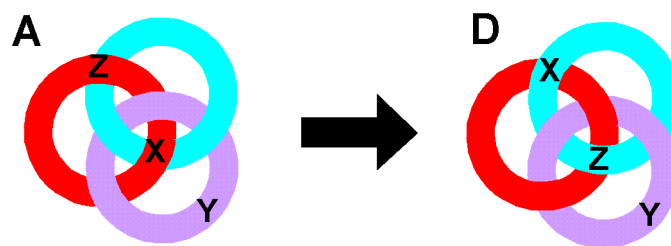
In fact turning over any of the rings in this way and passing them outside either of the other rings as you do so will always lead to arrangement C. And, of course, it works exactly the same way getting back to A.

What about arrangement D? Well if you take arrangement C, turn it over and swivel it around that's exactly what you end up with.



Just one question remains - is it possible to move directly between arrangements A and D? (And preferably without turning the whole arrangement over at any stage. ) The answer is, 'Yes!'

Here's how. Begin with arrangement A. This time hold the purple ring with your right hand at Y. With your left hand take hold of both the red and blue rings at X and pull them upwards and around until X ends up at Z. Effectively you have just turned two of the rings over together. The result - without any swivelling - is D.



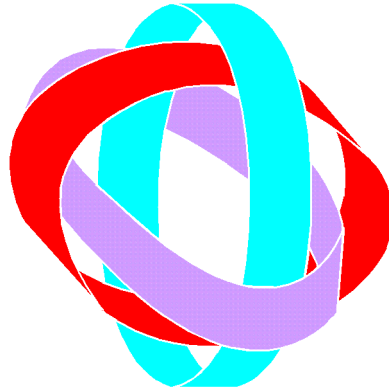
Do exactly the same again to return to arrangement A.

Of course, you could have turned over any pair of rings, either backwards or forwards, and the result - after a little swivelling - would have been the same. Six ways again.

You won't be surprised to learn that arrangement B can be transformed into arrangement C - and vice versa - in exactly the same way.

## Borromean Bands

It is possible to make Borromean objects in other ways. One of the easiest is to use bands instead of rings. Mathematically, the result is the same, but visually - and to the mind - the effect is very different.



This is because whereas the flat rings were simply laid over the top of each other, now the bands are inside and outside each other. This is equally implausible, of course. The blue strip is outside the red strip which is outside the purple strip which is outside the blue strip.

You will immediately want to know if the Borromean Bands can be manipulated into the same four different forms as the Borromean Rings - and the answer is, 'Yes and no!'

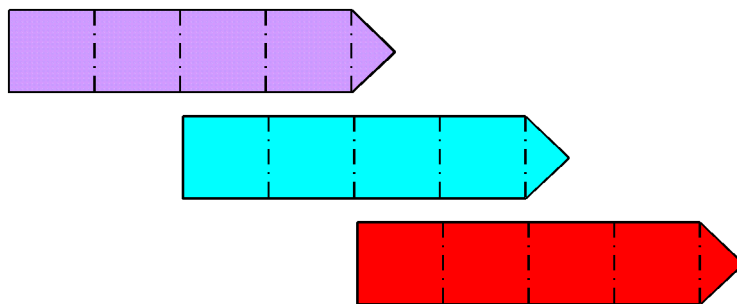
It may help to understand this if you think of the bands as lots of rings stacked on top of each other. Because the rings are so much thicker now the only way they can stay linked together is if they are allowed to twist sideways relative to each other. But this sideways twisted position is half-way between two of the flat arrangements - so now there are only two possible arrangements rather than four. Clear?



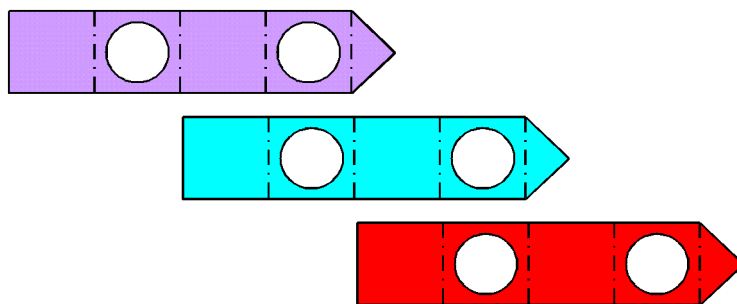
You can still transform one arrangement into the other - but it's not quite as easy with the bands as it was with the rings. Start by pulling the inside band out sideways - oh, sorry, I forgot, there is no inside band, What I mean is start by pulling the red band to the left until it's no longer inside the blue band. The purple band will get squashed flat as you do this. Now turn the red band over to the right so that it passes completely outside the blue band. Now take the purple band and turn that over to the right completely outside the red band. It will become twisted. Untwist it and the transformation is complete.

Of course, you could have chosen any coloured band to start with, and the same sequence - using any coloured band - will take you back to the first arrangement again.

The main interest of the Borromean Bands however lies in the fact that you can use them to make solid Borromean objects. If you make all three bands out of strips of four squares like the one shown here and link, or rather don't link, them together in the same way you get a Borromean Cube.

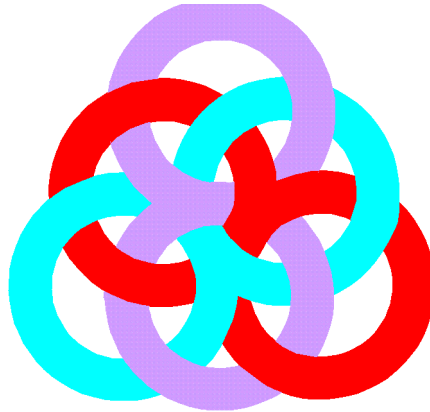


Cutting holes in the middle of each square allows the colour of the band below - or is it above? - to show through so that you can easily show that it's a genuine Borromean object.

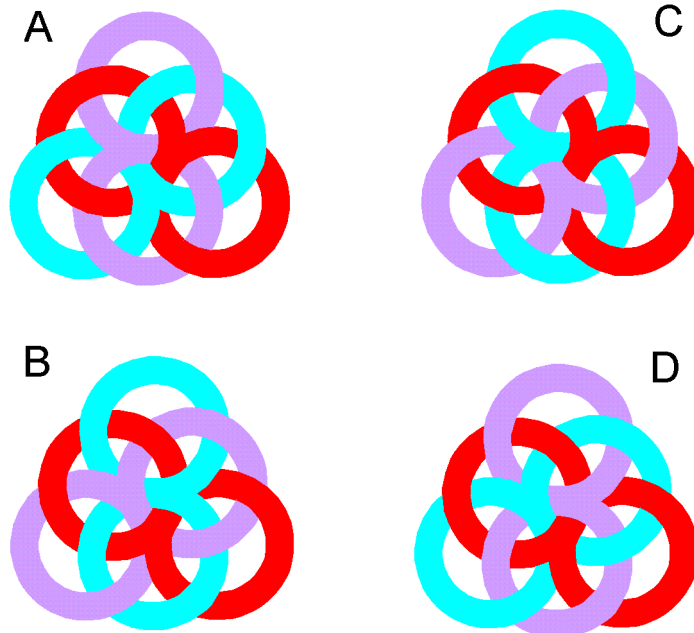


## Borromean Figures of Eight

You can easily join two rings together to form a figure of eight. Three eights of this kind will link, or rather not link, together to form another implausible Borromean object which looks like this:



You will see that, just as with the Borromean Rings, these Borromean Figures of Eight are not linked to each other but equally you cannot remove them from the design. Just as there were four different ways to arrange the Borromean Rings so there are four different ways to arrange the Borromean Figures of Eight.



It seems quite implausible that it should still be possible to transform these arrangements into each other, but it is still remarkably easy to do so.

A can still be transformed into B by turning it over and swivelling it around. C can be transformed into D in a similar manner.

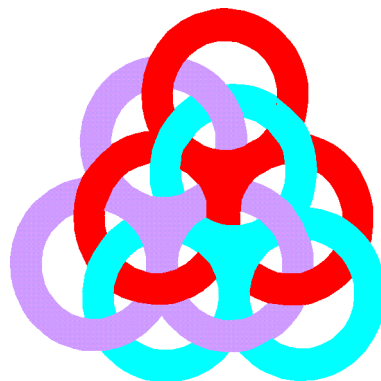
To transform A to C or B to D is harder. To achieve this choose any ring making up half of any eight. Perform exactly the same move required to transform A to C in the Borromean Rings - only this time you need to pass almost the whole of the rest of the arrangement through the ring as you turn it over. The ring will get twisted in the process. Untwist it and the transformation will be done.

The shortcut move from A to D is not possible with the Borromean Eights.

Another Borromean arrangement of three figures of eight is possible if the eights are bent in half so that one half of each eight can be linked in front and the other behind. This arrangement will not flatten.

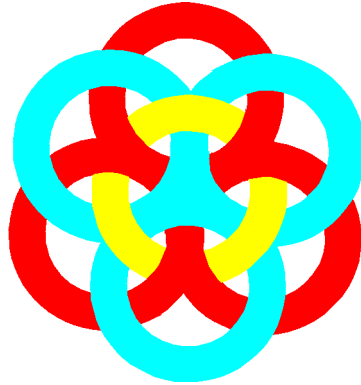
## Borromean Trefoils

Three rings can be linked together to form a trefoil. You will probably not be surprised to learn that three eights of this kind will link, or rather not link, together to form another implausible Borromean object which looks like this:





Alternatively you can arrange two trefoils and a single ring in a Borromean relationship like this.



As far as I know Borromean arrangements made using trefoils cannot be manipulated into their alternate forms.

It is possible that figures made from more than three rings can also be linked, or rather not linked, in Borromean relationships. I have not experimented to find out if this is the case.

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